Problem 1 — 1.8

In the morning section of a calculus course, 2 of the 9 women and 2 of the 10 men receive the grade of A. In the afternoon section, 6 of the 9 women and 9 of the 14 men receive an A. Verify that, in each section, a higher proportion of women than of men receive an A, but that, in the combined course, a lower proportion of women than men receive an A. Explain!

Problem 2 — 1.15

For what conditions on sets $A$ and $B$ does $A - B = B - A$ hold?

Problem 3 — 1.22 *

We have two identical glasses. Glass 1 contains $x$ ounces of wine; glass 2 contains $x$ ounces of water ($x \geq 1$). We remove 1 ounce of wine from glass 1 and add it to glass 2. The wine and water in glass 2 mix uniformly. We now remove 1 ounce of liquid from glass 2 and add it to glass 1. Prove that the amount of water in glass 1 is now the same as the amount of wine in glass 2.

Problem 4 — 1.27

Determine the set of real solutions $x$ to the inequality

$$\frac{|x|}{x + 1} \leq 1$$

Problem 5 — 1.29 *

Let $x, y, z$ be nonnegative real numbers such that $y + z \geq 2$. Prove that

$$(x + y + z)^2 \geq 4x + 4yz$$
Problem 6 — 1.32 *

Assuming only arithmetic (not the quadratic formula or calculus), prove that
\[ \{ x \in \mathbb{R} : x^2 - 2x - 3 < 0 \} = \{ x \in \mathbb{R} : -1 < x < 3 \} \]

Problem 7 — 1.36 *

Let \( S = [3] \times [3] \) (the Cartesian product of \{1, 2, 3\} with itself). Let \( T \) be the set of ordered pairs \((x, y) \in \mathbb{Z} \times \mathbb{Z}\) such that \(0 \leq 3x + y - 4 \leq 8\). Prove that \( S \subseteq T \). Does equality hold?

Problem 8 — 1.42

Let \( A = \{ \text{January, February, \ldots, December} \} \). Given \( x \in A \), let \( f(x) \) be the number of days in \( x \). Does \( f \) define a function from \( A \) to \( \mathbb{N} \)?

Problem 9 — 1.47 *

Let \( f : \mathbb{N} \times \mathbb{N} \to \mathbb{R} \) be defined by
\[ f(a, b) = \frac{(a + 1)(a + 2b)}{2} \]

(a) Show that the image of \( f \) is a subset of \( \mathbb{N} \).

(b) Determine exactly which natural numbers are elements of the image of \( f \). (Hint: Formulate a hypothesis by trying values.)

Problem 10 — 1.54

Let \( S = \{(x, y) \in \mathbb{R}^2 : y \leq x \text{ and } x + 3y \geq 8 \text{ and } x \leq 8\} \).

(a) Graph the set \( S \).

(b) Find the minimum value of \( x + y \) such that \( (x, y) \in S \). (Hint: On the graph from part (a), sketch the level sets of the function \( f \) defined by \( f(x, y) = x + y \).)
Problem 11 *

Let $r$ be a rational number and let $a$ and $b$ be irrational numbers. Which of the following numbers is necessarily irrational?

$$a + r \quad a + b \quad ar \quad ab \quad a^r \quad r^a \quad a^b$$

Prove your claims, either by proving that the number is irrational or by providing a counterexample. If you claim that a number is irrational, you should prove it.