Carnegie Mellon University
Department of Mathematical Sciences
21-122 Review Final Exam, Spring, 2006

Part 1

1. Integrate and simplify your result.

$$\int_0^{\pi/3} \cos^4 t \sin^2 t \; dt$$

2. Integrate

$$\int \frac{dx}{x^2\sqrt{9-x^2}}$$

3. Integrate

$$\int \frac{1}{x^2(x^2+4)} \; dx$$

4. Determine if the following integral converges or diverges, and evaluate the integral if it converges.

$$\int_0^{\infty} x^2 e^{-x^3} \; dx$$

Part 2

1. Find the gerneral solution of $\frac{dy}{dx} + \sin x + y\cos x = \cos x + y \sin x$.
2. Find the general solution of $\frac{dy}{dx} + \frac{1}{x}y = 1$.
3. Find the solution to $y'' - 8y' + 17y = 0$ that satisfy the conditions $y(0) = 1$, $y'(0) = -1$.
4. Suppose it takes 6 days for 25% of a radioactive substance to decay.
   (a) What is the half-life of the substance?
   (b) How long does it take for 75% of the substance to decay?

Part 3

1. Determine if the following series converges conditionally, converges absolutely, or diverges.

$$\sum_{n=1}^{\infty} \frac{(-\pi)^n}{n+n!}$$

2. Determine if the following series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n+1)}{1\cdot 4\cdot 7\cdots(3n+1)}$$

3. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \arctan\left(\frac{1}{n}\right)$$

4. Approximate the sum of the following series to within 0.1

$$\sum_{n=1}^{\infty} \frac{1}{n+4^n}$$

Part 4

1. Find the interval of convergence (*you must test endpoints, if applicable*) of the following series.

$$\sum_{n=0}^{\infty} 2^{-n}(2x-1)^n$$

2. Find a power series representation, and the interval on which it represents the function, for

$$f(x) = \frac{x^2}{4-x^2}$$

3. Find the Taylor series centered at $a = 8$ for

$$f(x) = \sqrt[3]{x}$$

4. Using Taylor's Inequality, determine an upper bound for the error in approximating $f(x) = e^{-2x}$ by its Taylor polynomial $T_2(x)$, centered at $a = 0$, on the interval $[-1/4, 1/4]$.