

Conic Sections and Quadric Surfaces Part II

Quadric Surfaces

In general, an eqn of a quadric surface is given by:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx +$$

$$Gx + Hy + Iz + J = 0$$

① Ellipsoid

② Elliptic Paraboloid

③ Hyperbolic Paraboloid

④ Hyperboloid ^{of one sheet}

⑤ Hyperboloid of two sheets.

⑥ Cone.

Intersect a quadric surface with a plane, what do you get?

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0$$

$$ax + by + cz + D = 0$$


$\Rightarrow z = \frac{-D - ax - by}{c}$ Plug in


$$A'x^2 + B'y^2 + D'xy + G'x + H'y + J = 0$$

Conic section.

Intersection of quadric surface with
a plane \longleftrightarrow conic section.


Quadric Surfaces


- $$1. \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



Ellipsoid
Cross sections are
Ellipses
- $$2. \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$



Elliptic Paraboloid
Cross sections are
Ellipses and parabolas
- $$3. \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

"Saddle"



Hyperbolic Paraboloid
Cross sections are
Hyperbolas and parabolas
- $$4. \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$


Hyperboloid of one
sheet
Cross sections are
Hyperbolas and Ellipses
- $$5. \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$


Hyperboloid of
two sheets
Cross sections are
Hyperbolas and Ellipses
- $$6. \frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$


Cone (Double)
Cross sections are
Ellipses and lines.