Exam I Review Problem Set
Math 21-259

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1. Calculate the length of the line segment AB and find the midpoint where A(2, 0, -1) and B(0, -1, 1).

2. Find an equation for the plane through (-5, -2, 6) that is parallel to the xy-plane.

3. Determine if the following equation represents a sphere and if so, find its center and radius: $x^2 + y^2 + z^2 + 10x + 4y - 12z + 56 = 0$.

4. Describe the region $\Omega = \{(x, y, z) : x^2 + y^2 \leq 25,\ -3 \leq z \leq 3\}$ in words.

5. Find α given that $3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$ and $\alpha\mathbf{i} - 36\mathbf{j} + 42\mathbf{k}$ are parallel.

6. Find all numbers x for which the angle between $\mathbf{c} = x\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{d} = \mathbf{i} + 1x\mathbf{j} + 3\mathbf{k}$ is $\frac{\pi}{3}$.

7. Find the scalar projection of the vector $\mathbf{a} = \mathbf{i} + \sqrt{3}\mathbf{k}$ onto the vectors $\mathbf{i},\ \mathbf{j},\ \mathbf{k}$.

8. Find the vector projection of the vector $\mathbf{a} = 2\,\mathbf{i}+\mathbf{j}-3\,\mathbf{k}$ onto the vector $\mathbf{b} = 4\,\mathbf{i}-2\,\mathbf{j}+\mathbf{k}$.

9. Calculate the following:

   (a) $\mathbf{j}\ .\ (\mathbf{2i} \times \mathbf{3k})$

   (b) $(\mathbf{i} - \mathbf{j} + 3\mathbf{k})\ .\ (\mathbf{2i} \times \mathbf{3k})$

   (c) $(\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \times (2\,\mathbf{i} + \mathbf{j})$

   (d) The volume of the parallelopiped with the given edges. $\mathbf{i}-2\mathbf{j}+\mathbf{k},\ 2\mathbf{j}-\mathbf{k},\ \mathbf{i}+\mathbf{j}-4\mathbf{k}$.

10. A vector parametrization for the line that passes through P(2, 4, 0) and is parallel to the line $\mathbf{r}(t) = (\mathbf{i} - \mathbf{j}) + t(3\mathbf{i} - \mathbf{k})$ is given by:

   (a) $\mathbf{r}(t) = 2\mathbf{i} + 4\mathbf{j} + t(\mathbf{i} - \mathbf{k})$
   (b) $\mathbf{r}(t) = 2\mathbf{i} + 4\mathbf{j} - t(3\mathbf{i} - \mathbf{k})$
   (c) $\mathbf{r}(t) = 2\mathbf{i} + 4\mathbf{j} + t(\mathbf{i} + \mathbf{k})$
   (d) $\mathbf{r}(t) = 2\mathbf{i} + 4\mathbf{j} + t(3\mathbf{i} + \mathbf{k})$
   (e) None of the above.

11. Give a vector parametrization for the line that passes through P(1, 2, -4) and is parallel to the line: $2(x-1) = 3(y-2) = 6(z+4)$

   (a) $\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k} + t(2\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k})$
   (b) $2\,\mathbf{i} - 6\,\mathbf{j} + 24\,\mathbf{k} + t(2\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k})$
   (c) $\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k} - t(3, \mathbf{i} + 2\,\mathbf{j} + \mathbf{k})$
   (d) $\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} + t(3\,\mathbf{i} - 2\,\mathbf{j} - \mathbf{k})$
   (e) None of the above.

12. Find a vector parametrization for the line segment that begins at (2, 7, -1) and ends at (4, 2, 3).

13. Determine whether the given two lines intersect and if they do, find the point and the angle of intersection.

$$l_1 : \mathbf{r}_1(t) = \mathbf{i} + t\,\mathbf{j}, \qquad l_2 : \mathbf{r}_2(t) = \mathbf{j} + u\,(\mathbf{i} + \mathbf{j})$$

14. Find the scalar parametric equation of the line that passes through P(1, 2, -4) and is parallel to the line: $l : \mathbf{r}_1(t) = \mathbf{i} + t\,\mathbf{j}$. How about the symmetric form?

15. Find the distance from the point P(1, 0, 2) to the line that passes through the origin and is parallel to $2\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$.

16. Find an equation of a plane that passes through the point P(2, 3, 4) and is perpendicular to $2\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$.

17. Find an equation of a plane that passes through the point P(1, 3, 1) and contains the line $l : x = t, y = t, z = -2 + t$.

18. Find an equation of a plane that passes through the points P(1, 0, 1), Q(2, 1, 0), R(1, 1, 1).

19. Find the angle between the planes $x - y + z - 1 = 0,\ 2x + y + z = -1$.

20. Find the distance from the point P(3, -5, 2) to the plane $8x - 2y + z = 5$.

21. Given an equation of a plane $x - 2y + z = 5$. Which of the following is true about the points A(2, 1, 1) and B(2, -1, 1)?

   (a) Both A and B lie on the same side of the plane

   (b) A and B lie of the opposite sides of the plane

   (c) Both the points A and B lie on the plane

   (d) The point A lie on the plane and B outside the plane

   (e) The point B lie on the plane and A outside the plane

22. Find the domain of the function $f(t) = t\,\mathbf{i} + \sqrt{(t+1)}\,\mathbf{j} - e^t\,\mathbf{k}$ and check for its continuity. Also, find $f'(t)$ and $\int_0^1 f(t)dt$.

23. Find $f(t)$ given that $f'(t) = \sin t\,\mathbf{i} + 3t^2\,\mathbf{j}$ and $f(0) = \mathbf{i} - \mathbf{k}$.

24. Find $lim_{t\to 0}3(t^2 - 1)\,\mathbf{i} + \cos t\,\mathbf{j} + \frac{t}{|t|}\,\mathbf{k}$.

25. Find the points on the curve $\mathbf{r}(t)$ at which $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ have opposite directions.

$$\mathbf{r}(t) = 5t\,\mathbf{i} + (3 + t^2)\,\mathbf{j}$$

26. Find a unit tangent vector and the principal normal vector(which is sometimes also referred as unit normal vector) to the curve $\mathbf{r}(t) = \cos(\pi t)\,\mathbf{i} + \sin(\pi t)\,\mathbf{j} + 3t\,\mathbf{k}$ at $t = 3$. Also, parametrize the tangent line and the normal line at the same indicated point. Lastly, find an equation of the osculating plane at the same point.

27. Find a point at which the following curves intersect. Also the find angle of intersection.

$$\mathbf{r}_1(t) = t\,\mathbf{i} + t^2\,\mathbf{j} - t^3\,\mathbf{k}, \qquad \mathbf{r}_2(u) = \sin 2u\,\mathbf{i} + u\cos u\,\mathbf{j} + u\,\mathbf{k}$$

28. Find the arc length of the curve $2\,\mathbf{i} + t^2\,\mathbf{j} + (t-1)^2\,\mathbf{k}$. from $t = 0$ to $t = 1$.

29. Find the radius of curvature of the curve $2y = x^2$.

30. Find the radius of curvature of the curve $\mathbf{r}(t) = 2t\,\mathbf{i} + t^3\,\mathbf{j}$ in terms of t.