Exam I Review Problem Set
Math 21-123

1. Find a formula for the general term of the following sequence:

   (a) $\{\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -1, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, ...\}$

   (b) $\{2, 7, 12, 17, ...\}$

   (c) $\{2, 1, 2, 1, ...\}$

2. Determine whether the following sequences converges or diverges. If it converges, find the limit.

   (a) $a_n = n \sin \frac{1}{n}$

   (b) $a_n = \frac{n}{\sqrt{n^3+1}}$

   (c) $a_n = (-1)^n (1 + \frac{1}{n})^n$

   (d) $a_n = \frac{10^n}{n!}$

   (e) $a_n = n^2 e^{-n}$

3. Determine whether the following sequences are monotonic or not. Give proper reasoning for your assertion.

   (i) $a_n = \ln(\frac{n}{n+1})$

   (ii) $a_1 = 1/4, a_n = \frac{n!}{2^n},\ n \geq 2$

(iii) $a_n = (3^n + 4^n)^{1/n}$

(iv) $a_n = (1 + \frac{1}{n})^n$, (HINT: $(1-x)^n \geq 1 - nx$ for all $x \leq 1$)

4. Determine whether the following sequences are bounded or not. Give proper reasoning for your assertion.

   (a) $a_n = (3^n + 4^n)^{1/n}$

   (b) $a_n = \frac{n}{e^n}$

   (c) $a_n = \frac{n!}{2^n}$

   (d) $a_n = n^{1/n}$

   (e) $a_1 = 1,\ a_n = 3 - \frac{1}{a_{n-1}},\ n \geq 2$

5. Find the limit of the sequence $\{\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, ...\}$.

6. Show that the sequence $\{a_n\}$ defined by $a_1 = 1/2, a_{n+1} = \frac{1}{2-a_n}$ satisfies $0 \leq a_n \leq 1$ and is decreasing. Deduce that the sequence is convergent and find its limit.

7. Determine whether or not the following series converge?

(a) $\sum(\frac{1}{k} - \frac{1}{k!})$

(b) $\sum \sin(\frac{\pi}{k^2})$

(c) $\sum(-1)^k(\sqrt{k+1} - \sqrt{k})$

(d) $\sum(-1)^k \frac{(k!)^2}{(2k)!}$

(e) $\sum \frac{\cos(\pi k/4)}{k^2}$

8. If the sum of the first six terms of a geometric series is 5.25 and the common ratio is $-\frac{1}{2}$, then find the first term of the series.

9. If the fourth term in geometric series is $\frac{4}{3}$ and the seventh term is $\frac{32}{81}$, then find the value of the common ratio.

10. Then find the sum of the first nine terms of the geometric series that has that has $a_4 = 48$ and $a_6 = 192$, where a_n is then n^{th} term of the geometric series $\sum_{n=0}^{\infty} ar^n$.

11. In the first stage of a chain email, four people send a message to four of their friends. Then what are the number of stages(to the nearest whole number) required for one million people to have received the email?

12. Estimate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ with an error < 0.005.

13. Ex 8.2 Problem 4

14. Ex 8.2 Problem 12

15. Ex 8.2 Problem 14

16. Ex 8.2 Problem 17

17. Ex 8.2 Problem 18

18. Ex 8.2 Problem 47(b)

19. Ex 8.2 Problem 50

20. Ex 8.3 Problem 23

21. Ex 8.3 Problem 24

22. Ex 8.3 Problem 25

23. Ex 8.3 Problem 27

24. Ex 8.4 Problem 8

25. Ex 8.4 Problem 24

26. Ex 8.4 Problem 27

27. Ex 8.4 Problem 42

28. Mark True or False.

(a) Every unbounded sequence is divergent.

(b) If $\lim_{n\to\infty} a_n = 0$ then $\sum a_n$ converges.

(c) If a_n converges then $\lim_{n\to\infty} a_n = 0$

(d) Every telescoping series converges.

(e) Every alternating series converges.

(f) Every absolutely convergent series is convergent.

(g) Every convergent series is absolutely convergent.

(h) If $\sum |a_n|$ is divergent then $\sum a_n$ is also divergent.

(i) If $\sum {a_n}^2$ is convergent then $\sum a_n$ is absolutely convergent.

(j) If $\sum {a_n}^2$ is convergent then $\sum a_n$ is convergent.

(k) If $\sum a_n$ is convergent then $\sum {a_n}^2$ is convergent.

(l) If $0 \le a_n \le b_n$ for every n and $\sum b_n$ diverges then $\sum a_n$ converges.

(m) The ratio test can be used to determine whether $\sum \frac{1}{n^2}$.

(n) Let a_n be a given series. If the sequence of partial sum is bounded then the series $\sum a_n$ converges.

(o) Let a_n be a given series of positive terms. If the sequence of partial sum is bounded then the series $\sum a_n$ converges.

(p) If $\{a_k\}$ is a decreasing sequence of positive numbers that converge to 0 then the alternating series $\sum(-1)^k a_k$ necessarily converge.