1. Find $f(t)$ given that $f'(t) = \sin t\,\mathbf{i} + 3t^2\,\mathbf{j}$ and $f(0) = \mathbf{i} - \mathbf{k}$.

2. Find $\lim_{t\to 0} 3(t^2-1)\,\mathbf{i} + \cos t\,\mathbf{j} + \frac{t}{|t|}\,\mathbf{k}$.

3. Find the points on the curve $\mathbf{r}(t)$ at which $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ have opposite directions.

$$\mathbf{r}(t) = 5t\,\mathbf{i} + (3+t^2)\,\mathbf{j}$$

4. Find a vector function that represents the curve of intersection of the two surfaces.

   (a) The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

   (b) The cone $z = \sqrt{x^2 + y^2}$ and the plane $z = x + 1$.

5. Find the point at which the following curves intersect. Also the find angle of intersection.

$$\mathbf{r}_1(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}, \qquad \mathbf{r}_2(u) = (1+u)\,\mathbf{i} + u^2\,\mathbf{j} + \frac{1}{8}\,\mathbf{k}$$

6. A particle moves so that $\mathbf{r}(t) = 2\,\mathbf{i} + t^2\,\mathbf{j} + (t-1)^2\,\mathbf{k}$. At what time is the speed a minimum?

7. Find the arc length of the curve $2\,\mathbf{i} + t^2\,\mathbf{j} + (t-1)^2\,\mathbf{k}$. from $t = 0$ to $t = 1$.

8. Find the position, velocity, acceleration vector at $t = 0$ of the particle moving along the curve $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + 2\sqrt{(2)}t\mathbf{k}$. Also, find the speed and the curvature of this curve at the same point. Reparametrize the given curve in terms of arc length starting at $t = 0$.

9. Interpret $\mathbf{r}(t)$ as the position of a moving object at time t. Determine the normal and tangential components of acceleration at time $t = 0$.

$$\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + 2\sqrt{2}t\mathbf{k}$$

10. Find the velocity and position vector given that $\mathbf{a}(t) = \mathbf{i} + 2\mathbf{j},\ v(0) = \mathbf{k}$, and $\mathbf{r}(0) = \mathbf{i}$.

11. Find the radius of curvature of the curve $2y = x^2$ at $x = 0$.

12. Find the curvature of the curve $\mathbf{r}(t) = 2t\,\mathbf{i} + t^3\,\mathbf{j}$ in terms of t.

13. Find the domain and range of the function $f(x, y, z) = \frac{z^2}{x^2 - y^2}$.

14. Find the domain and range of the function $f(x, y, z) = \frac{x+y+z}{|x+y+z|}$.

15. List equations of all 6 types of the quadric surfaces.

16. Identify the surface $9x^2 + 4y^2 - 36z^2 + 36 = 0$ and find its traces.

17. Sketch the cylinder $z = x^2$.

18. The surfaces $x^2 + y^2 + (z - 2)^2 = 2$ and $x^2 + y^2 = z^2$ intersect in a space curve C. Determine the projection of C onto the xy-plane.

19. Identify the level curves $f(x, y) = c$ and sketch the curves corresponding to the indicated values of c.

   (a) $f(x, y) = e^{xy}$ where $c = 0, 1, 2$.

   (b) $f(x, y) = \frac{x^2}{x^2+y^2}$ where $c = 0, 1/4, 1/2$.

   (c) $f(x, y) = \cos\sqrt{x^2 + y^2}$ where $c = 0, 1$.

   (d) $f(x, y) = \sin x$ where $c = 0, 1$.

20. Identify the level surfaces $f(x, y, z) = c$.

   (a) $f(x, y, z) = x + y + 3z$ where $c = 0$.

   (b) $f(x, y, z) = x^2 + y^2$ where $c = 4$.

21. Find an equation for the level curve of $f(x, y) = y^2 \arctan x$ that contains the point $P(1, 2)$.

22. Find $f_x(1, 2)$ and $f_y(1, 2)$ given that $f(x, y) = \frac{x}{x+y}$.

23. The intersection of a surface $z = x^2 + y^2$ and a plane $x = 2$ is a curve C in a space. Find equation for the line tangent to C at the point P(2, 1, 5).

24. Show that the function $g(x, y) = \frac{x^2 y}{x^4 + y^2}$ has limiting value 0 as $(x, y) \to (0, 0)$ along any line through the origin, but $\lim_{(x,y)\to(0,0)} g(x, y)$ still does not exist.

25. Find $\lim_{(x,y)\to(1,1)} \frac{x^2y}{x^4+y^2}$ if it exists.

26. Given

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & \text{when (x, y)} \neq (0,0), \\ 0 & \text{when (x, y)} = (0,0) \end{cases}$$

(a) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ if it exists.

(b) Is this function continuous everywhere? Give proper reasoning in support of your answer.

27. Given

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4} & \text{when (x, y)} \neq (0,0), \\ 0 & \text{when (x, y)} = (0,0) \end{cases}$$

(a) Find $f_x(0,0)$ and $f_y(0,0)$ if they exist.

(b) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ if it exists.

28. Evaluate $\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$ along:

(a) the line $y = mx$.

(b) the path $\mathbf{r}(t) = \frac{1}{t}\mathbf{i} + \frac{\sin t}{t}\mathbf{j},\ t > 0$.

29. Given

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{when (x, y)} \neq (0,0), \\ 0 & \text{when (x, y)} = (0,0) \end{cases}$$

Find $f_x(0,0)$ and $f_y(0,0)$. Is this function differentiable at (0, 0)? Is this function continuous at (0, 0)?

30. Given

$$f(x,y) = \begin{cases} \frac{x^2y^3}{2x^2+y^2} & \text{when (x, y)} \neq (0,0), \\ 0 & \text{when (x, y)} = (0,0) \end{cases}$$

(a) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ if it exists.

(b) Is this function continuous everywhere? Give proper reasoning in support of your answer.

31. Let f be a function of x and y with everywhere continuous second partials. Is it possible that
$$\frac{\partial f}{\partial x} = x + y \quad \text{and} \quad \frac{\partial f}{\partial y} = y - x?$$

32. The intersection of a surface $z = x^2 + y^2 + xy$ with a plane $y = 2$ is a curve in space. Find the equation of the tangent line to the curve C at the point $P(1, 2, 7)$.

33. Find the gradient of the function $f(x, y) = 2x + x^2 + 2y + y^2$ by using definition directly.

34. Find the gradient vector at the point P(2, 1) for the function $f(x, y) = ln(x^2 + y^2)$.

35. Find $\nabla[f(0,0)g(0,0)]$ given that $\nabla f(0,0) = \mathbf{i} - \mathbf{j}$, $\nabla g(0,0) = \mathbf{i} + \mathbf{j}$ and $f(0,0) = 1$, $g(0,0) = 2$.

36. Find the directional derivative of the function $f(x,y) = 2x^2 + 3y$ at the point P(1, 1) in the direction of the vector $2\mathbf{i} - 3\mathbf{j}$.

37. Determine the minimum directional derivative of $f(x, y) = 2x^2 + 3y$ at P(1, 1).

38. Find the directional derivative of the function $f(x, y) = 2x^2 + 3y$ at the point P(1, 1) towards the point (3, -2).

39. Find the directional derivative of the function $f(x, y, z) = x^2 + yz$ at the point P(1, -3, 2) in the direction of the path $r(t) = t^2\mathbf{i} + 3t\mathbf{j} + (1 - t^3)\mathbf{k}$.

40. Let $f(x, y) = 2x^2 + 3y^2$ represent the height of a mountain above the point (x, y). Let the positive x-direction point East and the positive y-direction point North. Suppose that I am standing above the point $P(3, 2)$ and sit a ball down. Which direction best represents the direction that the ball will start to roll?

   (a) Northwest

   (b) Northeast

   (c) Southwest

   (d) Southeast

   (e) None of the above

41. Find the rate of change of f with respect to t along the given curve.

$$f(x, y) = x - y,\ r(t) = at\mathbf{i} + b\cos at\mathbf{j}.$$

42. Let $h(t) = f(r(t))$ and $r(t) = \tan t\mathbf{i} + 2\sqrt{(2)}\sin t\mathbf{j}$. Given that $\nabla f(1, 2) = \mathbf{i} - \mathbf{j}$ and $\nabla f(2, 1) = \mathbf{i} + \mathbf{j}$, find $h'(\pi/4)$.

43. Answer the following conceptual questions.

(a) When is the directional derivative of f a maximum?

(b) When is it minimum?

(c) When is it zero?

(d) When is it half of its maximum value?

44. Let $f(x, y) = 2xy^2 - \frac{2y}{x}$ and the point P: (1, 2). Determine an equation for the tangent plane to the surface $z = f(x, y)$ at the point (1, 2, 4) on the surface. Also, find an equation of the normal line to the given surface at the indicated point.

45. Let $f(x, y) = 2xy^2 - \frac{2y}{x}$ and the point P: (1, 2).

(a) Calculate the directional derivative of f at the point P in the direction of the vector $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

(b) Determine a unit vector in the direction of the maximum directional derivative of f at P.

(c) Determine an equation for the tangent plane to the surface $z = f(x, y)$ at the point (1, 2, 4) on the surface.

46. Let $f(x, y)$ be the function given below so that $f(3, 0) = 9$. Give the differential approximation to $f(2.9, 0.01)$ where $f(x, y) = x^2e^{xy}$.

47. Find the linear approximation of the function $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at the point (2, 3, 4) and use it to estimate the number $(1.98)^3\sqrt{(3.01)^2 + (3.97)^3}$.

48. Find the linearization of the function $L(x, y) = \sqrt{x + e^{4y}}$ at the point (3, 0).

49. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

50. Find the second order partial derivative of $f(x, y) = 4x^3 - xy^2$.

51. If $v = x^2 \sin y + ye^{xy}$, where $x = s + 2t$ and $y = st$, use the Chain Rule to find $\partial v/\partial s$ and $\partial v/\partial t$ when $s = 0$ and $t = 1$.

52. Find $\partial z/\partial x$ and $\partial z/\partial y$ given that $x^5 + y^2 + z^3 + xy + zx + yz = \cos(x + y + z)$.

53. Use the chain rule to find partial derivatives $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$ when $u = 1,\ t = 1, s = 0$ given that

$$z = x^2 + xy^3,\ x = s^2t^3 + t^4u + 2u^2,\ y = t^3s + s^2t + u^3.$$

54. Determine whether or not the vector function is the gradient, $\nabla f(x, y)$, of a function everywhere defined. If so, find all the functions with that gradient.

$$(xe^{xy} + x^2)\mathbf{i} + (ye^{xy} - 2y)\mathbf{j}$$

55. Determine whether or not the vector function is the gradient, $\nabla f(x, y)$, of a function everywhere defined. If so, find all the functions with that gradient.

$$(y^2e^x - y)\mathbf{i} + (2ye^x - x)\mathbf{j}$$

56. Find the point(s) on the surface $z = xy$ at which the tangent plane is horizontal.

57. Find the stationary points and the local extreme values.

(a) $f(x, y) = x^2 - 2xy + 2y^2 - 3x + 5y$

(b) $f(x, y) = x^4 - 2x^2 + y^2 - 2$

58. Find the absolute extreme values taken on by $f(x,y) = 3 + x - y + xy$ on the set D where

(a) D is the closed triangular region with (-2, 2), (-2, 1), and (2, 1).

(b) $D = \{(x,y) |\ x^2 + y^2 \leq 4\}$.

(c) D is the region enclosed by $y = x^2$ and $y = 4$.

59. Find the maximum of $f(x, y) = x + y$ on the set where $x^4 + y^4 = 1$ and give the point where this occurs.

60. Find the points on the sphere $x^2 + y^2 + z^2 = 1$ that are closest and farthest from the point $(2, 1, 2)$.

61. A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth(the perimeter of a cross section perpendicular to the length) is at most 108 in. Find the dimensions of the package with largest volume that can be mailed.

62. Determine the absolute maximum and absolute minimum of

$$f(x,y) = x^2 + 4y^2 - 2x - 4y + 4$$

on the region D in the upper half-plane bounded by the x-axis and the ellipse

$$\frac{x^2}{4} + y^2 = 1.$$

63. Find the maximum value of $F(x,y,z) = 2x + 3y + 5z$ on the sphere

$$x^2 + y^2 + z^2 = 38.$$

64. A chemical company plans to construct an open (i.e., no top) rectangular metal tank to hold 256 cubic feet of liquid. It wants to determine the dimensions of the tank that will use the least amount of metal.

(a) Set up the problem in the form to use the method of Lagrange multipliers, i.e., minimize F subject to the constraint G = c.

(b) Determine the system of equations that has to be solved in order to solve the problem.