21-127 Concepts Homework 7: Solutions

1.8 The proportions with each grade are:

A grades	Men	Women
Morning	.20	.22
Afternoon	.64	.67
Total	.46	.44

This occurs because a higher proportion of men take the afternoon course, where the grading is much more generous. So although the women really do better in each class the men appear to do better overall; this is an example of Simpson's paradox.

9.4 As written the problem is trivial; $P(A \cup B) \geq P(A) > 1/2 > 0$. The problem should have asked you to show $P(A \cap B) > 0$. To do this we note,

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &\geq P(A) + P(B) - 1 \qquad \text{as } P(A \cup B) \leq 1 \\ &> \frac{1}{2} + \frac{1}{2} - 1 \\ &= 0 \end{aligned}$$

9.10 a) For a double-six we need the green die to be a 6 and the probability of this is $\frac{1}{6}$ (regardless of the result on the red die).

b) If we were shown one die, wlog the red one, and saw that it was a 6 then the probability is $\frac{1}{6}$ as in part (a). Alternatively, if we saw both dice and are calculating a conditional probability then,

$$\begin{aligned} P(\text{both } 6 | \text{at least one } 6) &= \frac{P(\text{both } 6)}{P(\text{at least one } 6)} \\ &= \frac{1/36}{11/36} \\ &= \frac{1}{11} \end{aligned}$$

9.18 Using Bayes' theorem,

$$
\begin{aligned}
P(\text{female}|\text{smoker}) &= \frac{P(\text{smoker}|\text{female})P(\text{female})}{P(\text{smoker}|\text{female})P(\text{female}) + P(\text{smoker}|\text{male})P(\text{male})} \\
&= \frac{(1/2)(1/3)}{(1/2)(1/3) + (1/3)(2/3)} \\
&= \frac{1/6}{7/18} \\
&= \frac{3}{7}
\end{aligned}
$$

9.23 Player A has a p chance of winning by a head on his first toss, together with a $(1-p)^2$ chance that both the first two tosses will be tails and then she is back to square one with a x chance of victory. So,

$$
\begin{aligned}
x &= p + (1-p)^2 x \\
\Rightarrow & [1-(1-p)^2]x = p \\
\Rightarrow & (2p - p^2)x = p \\
\Rightarrow & x = \frac{1}{2-p}
\end{aligned}
$$

Alternatively you can compute x by summing the infinite series of the probability player A wins on the first, third, fifth... toss.

For $p = \frac{1}{2}$ we have $x = \frac{1}{2-1/2} = \frac{2}{3}$

9.34 The number of different ways to get three red, two green and one blue face from six dice is the multinomial $\binom{6}{3\ 2\ 1} = \frac{6!}{3!2!1!} = 60$. The probability of each of these is $(1/2)^3(1/3)^2(1/6)$ so the overall probability is $\frac{60}{8.9.6} = \frac{5}{36}$.

Extra In the first case our state space is the possible ways to draw with order and with replacement, which has size 6^4. (We cannot just consider the ways to draw without order and with replacement as these are not equally likely.) Of these, 4! are rearrangements of $\{3,4,5,6\}$ and hence desired. So the probability of success is $\frac{4!}{6^4} = \frac{1}{54}$.

In the second case our state space is the ways to draw without order and without replacement (this is fine as the ways are all equally likely). It has size $\binom{7}{4}$ so the probability of success is $1/\binom{7}{4} = \frac{3!}{7.6.5} = \frac{1}{35}$.