### 1.1 Introduction to Proof: Quiz

**Theorem** Let $a$, $b$ be positive real numbers with $a < b$.
Then $\sqrt{a} < \sqrt{b}$.

Identify any mistakes in the following proofs.

**Proof 1**

$$\begin{aligned}\sqrt{a} < \sqrt{b} &\Leftarrow \sqrt{b} - \sqrt{a} > 0 \\ &\Leftarrow (\sqrt{b} - \sqrt{a})(\sqrt{b} + \sqrt{a}) > 0 \\ &\Leftarrow (\sqrt{b})^2 - (\sqrt{a})^2 > 0 \\ &\Leftarrow b - a > 0\end{aligned}$$

which is true

**Proof 2**
Suppose not

$$\begin{aligned}b \leq a &\Rightarrow b - a \leq 0 \\ &\Rightarrow (\sqrt{b})^2 - (\sqrt{a})^2 \leq 0 \\ &\Rightarrow (\sqrt{b} - \sqrt{a})(\sqrt{b} + \sqrt{a}) \leq 0 \\ &\Rightarrow \sqrt{b} - \sqrt{a} \leq 0 \\ &\Rightarrow \sqrt{b} \leq \sqrt{a}\end{aligned}$$

contradiction

**Proof 3**
Observe that for any positive $c$ and $d$ such that $c < d$ we have $c^2 < cd$ and also $cd < d^2$ and combining these we get $c^2 < d^2$. So

$$\begin{aligned}\sqrt{a} < \sqrt{b} &\Leftrightarrow (\sqrt{a})^2 < (\sqrt{b})^2 \\ &\Leftrightarrow a < b\end{aligned}$$