

1. Do there exist 1000 consecutive integers none of which is prime?
2. Write down carefully (without looking at your notes) a proof that there are infinitely many primes. By considering numbers of the form $4p_1p_2\dots p_k - 1$ prove that there are infinitely many primes of the form $4n - 1$. What would go wrong if we tried a similar proof to show there are infinitely many primes of the form $4n + 1$?