Teaching portfolio

Clive Newstead

Contents

Section 1

Reflections

Section 1.1

Statement of teaching philosophy

When teaching mathematics at the undergraduate level, it is my aim to help students make the transition from school-level mathematics, where the focus is primarily on computation and 'showing your work', to graduate-level and professional mathematics, where the focus shifts to finding solutions to new problems and expounding original theoretical work.

In particular, I aim to help my students make transitions in the following areas:

(1) **Communication:** from showing steps in a calculation, to writing proofs, making arguments and discussing ideas.

(2) **Independence:** gaining the confidence and mathematical maturity required to apply their knowledge and skills to solve new problems;

(3) **Personalisation:** from studying mathematics in a vacuum, to tailoring their mathematical education according to their own needs and interests.

What follows is a discussion of how I incorporate these three aspects into my teaching.

Communication: *writing and speaking mathematics*

Mathematics graduates, whether in industry or further study, are expected to be able to interpret and explain technical ideas, so I see it as my responsibility to incorporate communication skills into my course design and classroom teaching.

To help my students in the mathematics major develop these skills, I incorporate writing and discussion activities in my classes. For example, in one Math 300 *Foundations of Higher Mathematics* class, students wrote proofs of simple results in pairs and then volunteered their proofs for a class-wide critique and revision. I assess my Math 300 students' proof-writing skills in the form of a final project, in which students use LaTeX to write a self-contained document expounding a theorem of their choice.

As another example, in a recent Math 290 class on linear algebra, I assigned each student a 2×2 matrix; students then paired up, described their matrix geometrically to their partner, and were required to determine whether or not their matrices commuted with each other. This encouraged students to reason verbally about the geometric ideas from linear algebra, helping them to break away from purely algebraic pen-on-paper reasoning.

For non-mathematics majors, the focus is more on applications of mathematical tools. When teaching non-majors, I help students to interpret real-world information mathematically and then communicate their mathematics results in the context of the original setting. For example, in Math 211 *Short Course in Calculus*, each student wrote a blog post about an application of calculus to a field of their interest, and students commented on each others' posts. In their posts, students translated real-world problems, such as finding an appropriate dosage of a drug or determining elasticity of demand, into pure calculus problems. They applied the calculus techniques from the course, and then explained what their solution meant in terms of their original problem.

Independence: *gaining confidence and mathematical maturity*

Students in mathematics courses must develop the ability to identify solutions to new problems and solve them. I try to help my students gain the independence and confidence required to achieve this goal.

For example, when I teach computational classes such as calculus or linear algebra, I structure my classes with worksheets that are distributed to students at the beginning of class. After a brief introduction of a new concept, I give students some time to work on problems related to that concept individually or in pairs. We then work through the problem as a class, and I answer questions as they arise. This helps students to gain independence by providing them with early, low-stakes opportunities to put the course concepts into action.

When I taught Math 211 *Short Course in Calculus*, most of my students were juniors and seniors who had not taken a mathematics class since high school. I repeatedly checked in with students to see how they were finding the pace of the class and how confident they felt with the course concepts, and was able to adapt the course to match their needs.

I have found that office hours present an excellent opportunity for helping students to gain confidence in their mathematical skills, so I make a special effort to be approachable to students and encourage them to speak to me and their TAs about the course material.

Personalisation: *connecting mathematics with students' interests and goals*

One of the most challenging aspects of teaching mathematics is helping students to see what they are learning as interesting, useful and relevant to them. Doing so is essential to keeping them motivated to learn, and ultimately to succeed.

When designing a course, if the curriculum permits, I try to build in some flexibility in what topics to cover. This allows me to poll the class and find out what they are interested in.

For instance, when I teach Math 300 I allow students to choose the topic to be covered after the midterm—students vote on their preferences, and the most popular topic gets covered. In Math 290 there is less flexibility, so I incorporate this flexibility into the examples I choose to illustrate the topics from calculus and linear algebra.

I have found incorporating project work into my courses to be a great way for students to connect the course material with their interests.

Students in Math 300, for example, choose their own project topics, allowing them to preview material in more advanced courses that they might not have seen yet. For the Math 211 blog, students were able to choose the topic of their blog post; I met with each student individually to discuss their topic, and directed them towards references and supplementary materials.

When teaching a course with a large enrolment, this degree of customisation is not always possible. In these cases, I try to get a sense of what students' interests are—for example, by finding out their majors or their career goals—and incorporate references to these interests throughout the course.

Section 1.2

Statement on inclusive teaching in mathematics

In this statement, I describe some of the approaches I take, both inside and outside of the classroom, in order to improve the accessibility of mathematics in higher education.

Universal course design: *accessible materials and equitable assessments*

When I design a course, I do so with a heterogeneous class of students in mind in order to direct my selection of assessment methods, choice of course materials, and syllabus design.

Foundations of Higher Mathematics (Math 300) at Northwestern, for example, is historically lecture-based and assessed solely through problem sheets and high-stakes examinations. I redesigned the course from scratch to incorporate project-based work, frequent opportunities for low-stakes practice prior to higher-stakes assessments, and a variety of in-class activities. Doing so increased the accessibility of my course to all of my students, and had the additional beneficial effect that I was able to target specific learning objectives in each kind of assessments.

A high priority of mine is to ensure that students are not disadvantaged if they cannot afford to spend money on expensive textbooks or class equipment. This was one of the main motivations I had when I wrote my textbook, which I keep freely available online. In my coordinated linear algebra and calculus courses, I write handouts containing summary notes and worked examples, in order to ensure that all examinable material is freely available to my students even if they do not have the required text. I also try to avoid using expensive equipment and software, such as iClickers and WebAssign, favouring free alternatives, such as the MAA's open-source WeBWorK.

Inclusive teaching strategies: *active learning and classroom climate*

There are many considerations I make in the classroom in order to improve accessibility, maintain a healthy classroom climate, and benefit from student diversity.

Research has shown that active learning techniques improve student learning gains when compared with traditional lecture, with an especially pronounced effect for students from underrepresented groups and nontraditional educational backgrounds. As such, I make heavy use of active learning methods in my teaching, ranging from individual or group problem-solving activities, to class-wide critiques of written proofs, and even debates on matters such as whether zero is a natural number.

In order to foster an inclusive classroom climate, I build a rapport with my students so that they perceive me as approachable and supportive. At a basic level, I make an effort to be enthusiastic, to learn students' names and to take an interest in their lives. Additionally, communicate not just my high expectations, but also my confidence that students can achieve them—in doing so, I try to encourage a so-called *growth mindset* in my students, which boosts their confidence and helps them to become self-directed learners.

Outside the classroom: *approachability, outreach and recruitment*

Students' lives and backgrounds influence their learning, motivation and success. As such, I find it important to take an interest in my students as individuals that exist outside of the classroom, and to engage in outreach activities in order to improve access to mathematics in higher education for the students of the future.

A cornerstone of my teaching is my approachability to students and willingness to support them. If I notice a student is struggling or is absent, I reach out to them to check on their well-being and offer additional support if needed. I have met with struggling students individually, sometimes even on evenings and weekends, to help them get back up to speed and regain confidence in the class. This support continues even after my course is over—to this day, I am often visited or contacted by students who I taught several years ago, even those who have graduated, who come to me to discuss their current classes, career options and sometimes even personal issues.

Outreach is a particularly important aspect of improving the accessibility of mathematics. I am currently involved in facilitating the Northwestern Emerging Scholars Program (NESP), which is aimed at improving recruitment of women and underrepresented groups in the mathematics major. Students enrolled in NESP meet once per week to explore concepts from upper-level mathematics courses in a session led by a peer leader. The goal of the program is to help students develop a sense of belonging in the mathematical community and to make an informed decision about whether to pursue mathematics further. Last quarter, the NESP was awarded a Daniel I. Linzer Grant for Innovation in Diversity and Equity in the amount of $3800. I would be interested in piloting a similar program at another institution.

In terms of outreach in the wider community, for a year and a half I volunteered for a free after-school program at *Assemble* in Pittsburgh, a local community organisation in one of Pittsburgh's less affluent neighbourhoods—in this role, I helped 8–14 year-olds with their mathematics homework and assisted with hands-on STEM-based projects, as well as tabling at other community events. I thoroughly enjoyed this experience, and I hope to continue community outreach activities in the future.

References

[A+10] Susan A. Ambrose et al. *How learning works: Seven research-based principles for smart teaching*. John Wiley & Sons, 2010.

[CH09] Roxanne Cullen and Michael Harris. Assessing learner-centredness through course syllabi. *Assessment & Evaluation in Higher Education*, 34(1):115–125, 2009.

[F+14] Scott Freeman et al. Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, 111(23):8410–8415, 2014.

Section 2

Experience and contributions

Section 2.1

Teaching experience and professional development

Classroom teaching experience

The following table summarises my past, current and scheduled teaching. 'Math xxx-x' denotes a course at Northwestern, and '21-xxx' or '15-xxx' denotes a course at Carnegie Mellon.

Code	Course title	Semester(s)/Quarter(s)
Math 211-0	*Short Course in Calculus*	(Sp19)
Math 230-B	‡*Multivariable Integral Calculus for SPS*	(Wi20)
Math 290-1,2,3	†*Linear Algebra & Multivariable Calculus*	AY18–19, AY19–20
Math 300-0	*Foundations of Higher Mathematics*	Fa18, Wi19
Math 300-CN	‡*Foundations of Higher Mathematics for SPS*	(Sp20)
Math 330-1,2,3	†*Abstract Algebra*	AY19–20
Math 330-A	‡*Abstract Algebra for SPS*	Fa19
Math 399-0	Independent study project & thesis	Fa19, (Sp20)
15-151	*Concepts of Mathematics for the SAMS program*	Su17
21-127	*Concepts of Mathematics for the AP/EA program*	Su15
21-256	*Multivariate Analysis*	Su14

†: full year course sequence (Fa,Wi,Sp). ‡: evening course taught for the School of Professional Studies (SPS).

This academic year (2019–20) I am serving as course coordinator Math 290.

Professional development

In addition to the experience gained from teaching a variety of courses, I actively seek professional development opportunities in order to improve as a teacher and further my contributions to education. Some of these are described below.

Searle Teaching-as-Research Program. In Spring 2019 designed and implemented a blogging assignment for Math 211 *Short Course in Calculus*, and conducted a study to assess whether it helped students to engage with the calculus material.

Eberly Future Faculty Program. In December 2015, I completed the *Future Faculty Program*, which had four components:

- *Seminars.* I attended 11 two-hour seminars on teaching and learning, on topics including motivation and engagement, teaching first-year undergraduates, providing helpful feedback and supporting students' academic integrity.
- *Teaching feedback consultations.* I attended a *Microteaching Workshop*, a small-group collaborative feedback session in which I have a five-minute mini-lecture and received feedback; and I had a comprehensive classroom observation of my teaching for 21-127 *Concepts of Mathematics* in summer 2015.

- *Course and syllabus design project.* For this requirement, I developed a syllabus for 21-256 *Multivariate Analysis*, a multivariable calculus and linear algebra class tailored for undergraduates majoring in economics and business.
- *Individualised project.* I developed a course portfolio for 21-127 *Concepts of Mathematics*, including a set of course notes (which have now evolved into a textbook), lesson plans, in-class exercise sheets, homework problem sets, supplemental handouts, and slides.

A more detailed description of the Future Faculty Program, and documentation of my completion, can be found in Section 5, starting at page 30.

Graduate Teaching Fellow Program. I worked as a Graduate Teaching Fellow ('GTF') for the Eberly Center for Teaching Excellence and Educational Innovation from January 2015 until August 2018, promoted to Senior Graduate Teaching Fellow in January 2017.

In addition to the teaching consultations and services I provided to other graduate students and postdocs (see Section 2.2), this role continued my professional development in several ways, including Teaching Circles, which are regular meetings with other GTFs to discuss research papers on teaching and learning

Conferences and workshops. I have attended several conferences and workshops relating to teaching and learning (and presented at two), including the POD Network Conference in 2015, two Teaching and Learning Summits at Carnegie Mellon, and special sessions on teaching and learning at the Joint Mathematics Meeting in 2018.

Section 2.2

Contributions to education

This section highlights some of the contributions I have made to the field of education outside of the classroom.

Textbook

Originating as lecture notes I wrote for 21-127 *Concepts of Mathematics* in Summer 2015, I have written a textbook entitled *An Infinite Descent into Pure Mathematics*, aimed at helping readers acquire the skills and knowledge required to study pure mathematics at an advanced level.

It is currently being used to teach 21-128 *Mathematical Concepts and Proofs* at Carnegie Mellon University, Math 300 *Foundations of Higher Mathematics* at Northwestern University, and Math 301 *Introduction to Proofs* at Johns Hopkins University.

The book is freely available online (`https://infinitedescent.xyz/`) and I am currently preparing it for publication.

More information about the book can be found in Section 4.

Course materials

In addition to the textbook, I have generated a variety of course materials. The following is a list of the kinds of materials I have created, together with a page reference to a sample provided in Section 5 of this portfolio.

- **Syllabi** — page 36 (Math 300 Wi19)
- **Grading rubrics** — page 46 (Math 290 Fa18) and page 47 (Math 300 Wi19)
- **Class handouts** — page 49 (Math 290 Fa18)
- **Lesson plans** — page 53 (15-151 Su17)
- **Homework assignments** — page 54 (15-151 Su17)
- **Projects** — page 56 (Math 300 Wi19) and page 59 (Math 211 Sp19)
- **Examinations** — page 62 (21-256 Su14) and page 65 (Math 300 Fa18)

Training sessions

- **Active learning training sessions**, *August 2015 and August 2017*.

Co-facilitated TA training sessions for the Mellon College of Science, focussing on using active learning techniques for teaching recitations in mathematics, physics, chemistry and biology.

- **Departmental TA training**, *August 2014, August 2015, August 2016*.

  Led microteaching sessions for first-time teaching assistants in mathematics. This included recording the TAs, providing verbal and written feedback, and soliciting feedback from other participants.

Teaching consultations

In my role as Graduate Teaching Fellow at the Eberly Center from 2015–2018, I provided consultations with graduate students and postdocs on various aspects their teaching. A summary of the consultations I provided is as follows:

- **Classroom observations** (25 total).

  A typical classroom observation consultation has four stages: a pre-observation meeting to discuss the client's class plans and desired feedback; the class observation, where I attend and make comprehensive notes on the client's teaching strategies; a post-observation meeting, where the client discuss the class and my feedback; and the observation memo, which is a detailed written report.

- **Statements of teaching philosophy** (12 total).

  For statement of teaching philosophy consultations, I meet with clients to discuss ideas for their statements, and then provide feedback on iterative drafts. The feedback is structured according to a rubric.

- **Microteaching workshops** (5 total).

  In a microteaching workshop, a group of 3–6 graduate students and postdocs each deliver a five-minute teaching sample, and share feedback on each other's teaching. My role is to facilitate the discussion, video record the teaching samples, and meet with attendees individually to discuss the recording.

In addition to these kinds of consultations, I contributed to the design and facilitation of workshops on teaching and learning, and I helped to train new Graduate Teaching Fellows.

Conference talks

- **CMU Teaching and Learning Summit**, *14th October 2016*.

  Presented a 'quick-fire talk' entitled *Engaging students in mathematics through active learning*. A video recording of the talk is available from the following URL:

  https://www.cmu.edu/teaching/summit/overview/QFT_danforth_4.mp4

- **POD 2015 conference**, *4th–8th November 2015*.

  Co-designed and co-facilitated a session entitled *TAs Serving TAs: A roundtable for graduate teaching consultants*. The discussion focussed on challenges facing graduate students in teaching consultant roles and solutions to such problems.

Miscellaneous contributions

- **Classroom Renovation Working Group**, *Fall 2017 & Spring 2018*

  As part of a university-wide classroom renovation project at Carnegie Mellon, I collected data on how classrooms were being used. I observed 12 classes and used a spreadsheet to make note of all interactions between people and the classroom, such as moving desks or writing on a blackboard, and to provide qualitative data concerning the obstacles presented by the classroom, such as noisy vents or poor lighting. This data was then used by a university working group to inform decisions on which classrooms should be renovated and what renovations should be made.

- **Diagnostic examinations**, *Summer & Fall 2017*

  I helped design two diagnostic exams, which were used to determine which students were ready be placed in more advanced mathematics classes. The classes in question were:

  - 21-242 *Matrix Theory*, an advanced, proof-based linear algebra course; students who did not place into this class instead took 21-241 *Matrix Algebra and Linear Transformations*.
  - 15-151 *Mathematical Concepts and Proofs*, the proof-based mathematics class taught in the CMU Summer Academy for Mathematics and Science; students who did not place into this class instead took calculus.

Section 3

Evidence of effectiveness in teaching

Section 3.1

Teaching awards

In April 2016, I was presented with two separate teaching awards: the *Carnegie Mellon University Graduate Student Teaching Award*, and the *Hugh D. Young Graduate Student Teaching Award*. The former is a university-wide award, and the latter is an award within the Mellon College of Science. I was nominated for both awards by the Department of Mathematical Sciences. Descriptions of these awards are below.

Carnegie Mellon University Graduate Student Teaching Award

The Graduate Student Teaching award is open to all Carnegie Mellon University graduate students, and is awarded annually to recognise a graduate student who has demonstrated teaching excellence at the university. The award recipient is selected by the Graduate Student Teaching Award Committee, which consists of three faculty members, three graduate students, two undergraduate students and a non-voting chairperson from the Eberly Center for Teaching Excellence and Educational Innovation.

More information: `https://www.cmu.edu/celebration-of-education/graduate-student/teaching-nomination-process.html`

Press release: `https://www.cmu.edu/mcs/news-events/2016/0425-newstead-gradaward.html` (see page 27)

Hugh D. Young Graduate Student Teaching Award

The Hugh D. Young Graduate Student Teaching Award is awarded annually to recognise effective teaching by a graduate student in the Mellon College of Science. Each department in the college may nominate one graduate student per year. The recipient of the award is chosen by a committee appointed by the Dean of MCS, that includes an MCS Associate Dean and if possible, the Associate Deans for Undergraduate Affairs from the Carnegie Institute of Technology, and the School of Computer Science, as well as two MCS faculty members.

More information: `https://www.cmu.edu/mcs/people/faculty/resources/handbook/hughYoung.html`

Press release: `https://www.cmu.edu/mcs/news-events/2016/0504-mcs-edu-res-awards.html`

Section 3.2

Student evaluations

This section summarises the student evaluations collected at the end of each course that I taught as a course instructor and as a teaching assistant, with the exception of 21-127 *Concepts of Mathematics* in Summer 2015 (for which evaluations were not collected). Averages are provided where available.

Course and Teaching Evaluations (CTEC) — Northwestern University

All CTEC scores are on a scale of 1 (very low) to 6 (very high). Original reports and student comments are available on request.

Averages at the department, college and university level are not officially released, so are not included. I have been told that averages in the mid 4.xx range are typical.

Math 211 — Spring 2019 — Short Course in Calculus

	Mean	Median	Std Dev
Overall rating of the instruction	**5.80**	**6**	**0.45**
Instructor stimulates interest in subject	5.60	6	0.89
Instructor was prepared for class	6.00	6	0.00
Instructor communicated course content and ideas	5.80	6	0.45
Instructor's enthusiasm in teaching this class	5.80	6	0.45
Overall rating of the course	**5.80**	**6**	**0.45**
How much you learned in the course	5.60	6	0.55
Effectiveness of course in challenging you	5.60	6	0.55
Rating of instructional materials used	5.60	6	0.55
Response rate: 63% (5 of 8)			

Math 290-3 — Spring 2019 — Linear Algebra and Multivariable Calculus (part 3 of 3)

	Mean	Median	Std Dev
Overall rating of the instruction	**5.71**	**6**	**0.47**
Instructor stimulates interest in subject	5.59	6	0.62
Instructor was prepared for class	5.76	6	0.44
Instructor communicated course content and ideas	5.76	6	0.44
Instructor's enthusiasm in teaching this class	5.94	6	0.24
Overall rating of the course	**5.06**	**5**	**0.83**
How much you learned in the course	5.53	6	0.62
Effectiveness of course in challenging you	5.59	6	0.80
Rating of instructional materials used	4.94	5	0.97
Response rate: 85% (17 of 20)			

Math 300 — Winter 2019 — Foundations of Higher Mathematics

	Mean	Median	Std Dev
Overall rating of the instruction	**5.89**	**6**	**0.33**
Instructor stimulates interest in subject	5.56	6	0.53
Instructor was prepared for class	6.00	6	0.00
Instructor communicated course content and ideas	5.89	6	0.33
Instructor's enthusiasm in teaching this class	6.00	6	0.00
Overall rating of the course	**5.33**	**6**	**0.87**
How much you learned in the course	5.22	5	0.67
Effectiveness of course in challenging you	5.33	6	0.87
Rating of instructional materials used	5.78	6	0.44
Response rate: 69% (9 of 13)			

Math 290-2 — Winter 2019 — Linear Algebra and Multivariable Calculus (part 2 of 3)

	Mean	Median	Std Dev
Overall rating of the instruction	**5.63**	**6**	**0.62**
Instructor stimulates interest in subject	5.94	6	0.25
Instructor was prepared for class	5.75	6	0.45
Instructor communicated course content and ideas	5.44	6	0.81
Instructor's enthusiasm in teaching this class	6.00	6	0.00
Overall rating of the course	**5.19**	**5**	**0.66**
How much you learned in the course	5.63	6	0.50
Effectiveness of course in challenging you	5.75	6	0.45
Rating of instructional materials used	4.88	5	0.81
Response rate: 80% (16 of 20)			

Math 300 — Fall 2018 — Foundations of Higher Mathematics

	Mean	Median	Std Dev
Overall rating of the instruction	**5.86**	**6**	**0.38**
Instructor stimulates interest in subject	5.71	6	0.76
Instructor was prepared for class	6.00	6	0.00
Instructor communicated course content and ideas	6.00	6	0.00
Instructor's enthusiasm in teaching this class	5.86	6	0.38
Overall rating of the course	**5.57**	**6**	**0.79**
How much you learned in the course	5.43	6	0.79
Effectiveness of course in challenging you	5.57	6	0.53
Rating of instructional materials used	5.57	6	0.79
Response rate: 88% (7 of 8)			

Math 290-1 — Fall 2018 — Linear Algebra and Multivariable Calculus (part 1 of 3)

	Mean	Median	Std Dev
Overall rating of the instruction	**5.53**	**6**	**0.64**
Instructor stimulates interest in subject	5.33	6	0.82
Instructor was prepared for class	5.87	6	0.35
Instructor communicated course content and ideas	5.60	6	0.51
Instructor's enthusiasm in teaching this class	5.80	6	0.41
Overall rating of the course	**5.20**	**5**	**0.56**
How much you learned in the course	5.73	6	0.46
Effectiveness of course in challenging you	5.60	6	0.83
Rating of instructional materials used	5.13	5	0.83
Response rate: 94% (15 of 16)			

Faculty Course Evaluations (FCE) — Carnegie Mellon University

All FCE scores are on a scale of 1 (poor) to 5 (excellent). Averages at the department, college and university level are provided. Original reports and student comments are available on request.

15-151 — Summer 2017 — Concepts of Mathematics for the SAMS program

	Average	Std Dev	CSD	SCS	CMU
Response rate: 82% (23 of 28)					
Overall teaching rate	**4.74**	**0.45**	**4.50**	**4.36**	**4.29**
Overall course rate	**4.26**	**0.86**	**4.51**	**4.33**	**4.24**
Interest in student learning	4.91	0.29	4.57	4.48	4.41
Clearly explain course requirements	4.78	0.52	4.58	4.39	4.32
Clear instructor goals & objectives	4.83	0.49	4.60	4.43	4.36
Instructor provides feedback to students	4.39	0.94	4.34	4.28	4.19
Demonstrate importance of subject matter	4.48	0.85	4.57	4.46	4.41
Explains subject matter of course	4.65	0.57	4.54	4.37	4.31
Shows respect for all students	4.91	0.42	4.72	4.60	4.60

Faculty Course Evaluations for 21-127 Concepts of Mathematics in Summer 2015 were not collected by the department as all students were enrolled in the AP/EA pre-college program.

21-256 — Summer 2014 — Multivariate Analysis

	Average	Std Dev	Math	MCS	CMU
Response rate: 100% (6 of 6)					
Overall teaching rate	**5.00**	**0.00**	**4.09**	**4.39**	**4.44**
Overall course rate	**5.00**	**0.00**	**4.02**	**4.29**	**4.44**
Interest in student learning	4.83	0.41	4.30	4.57	4.57
Clearly explain course requirements	5.00	0.00	4.09	4.37	4.43
Clear instructor goals & objectives	5.00	0.00	4.13	4.34	4.46
Instructor provides feedback to students	4.67	0.52	4.03	4.16	4.26
Demonstrate importance of subject matter	4.83	0.41	4.03	4.38	4.52
Explains subject matter of course	4.83	0.41	4.19	4.38	4.45
Shows respect for all students	5.00	0.00	4.59	4.70	4.63

TA evaluations — Carnegie Mellon University

All TA evaluation scores are on a scale of 1 (poor) to 5 (excellent).

21-128 — Fall 2016 — Mathematical Concepts and Proofs

	Sec A	Sec B	Sec C	Dept	MCS
Response rate	86%	87%	68%	63%	61%
The TA was consistently well-prepared and organized for class	5.00	5.00	5.00	4.23	4.34
The TA made me feel free to ask questions	5.00	4.92	4.83	4.34	4.35
The TA clearly and effectively responded to students' questions and comments	5.00	4.96	5.00	4.09	4.20
The TA treated all students with respect	5.00	5.00	4.94	4.60	4.60
The TA's English was understandable	5.00	4.96	4.89	4.35	4.46
The TA was knowledgeable and familiar with concepts covered in lectures	5.00	5.00	5.00	4.35	4.45
The TA encouraged students to participate actively in recitation	4.92	4.85	4.89	4.35	4.15
The TA managed class time effectively	4.96	4.96	4.94	4.06	4.21
The TA was readily available for consultation with students	4.92	4.96	5.00	4.22	4.33
Overall, the quality of this recitation was:	**5.00**	**4.96**	**5.00**	**4.34**	**4.20**
Overall, the TA was effective in his/her role	**5.00**	**4.96**	**5.00**	**4.14**	**4.26**

21-128 — Fall 2015 — Mathematical Concepts and Proofs

	Sec A	Sec B	Sec C	Dept	MCS
Response rate	93%	68%	93%	59%	58%
The TA was consistently well-prepared and organized for class	4.96	5.00	5.00	4.18	4.07
The TA made me feel free to ask questions	4.81	4.89	4.96	4.22	4.17
The TA clearly and effectively responded to students' questions and comments	4.92	5.00	4.96	4.07	3.96
The TA treated all students with respect	4.85	5.00	5.00	4.47	4.42
The TA's English was understandable	4.96	4.96	4.96	4.35	4.30
The TA was knowledgeable and familiar with concepts covered in lectures	4.96	5.00	5.00	4.43	4.25
The TA encouraged students to participate actively in recitation	4.85	4.84	4.85	3.96	3.95
The TA managed class time effectively	4.88	4.95	5.00	4.13	3.97
The TA was readily available for consultation with students	4.88	5.00	4.00	4.25	4.12
Overall, the quality of this recitation was:	**4.96**	**5.00**	**4.96**	**4.08**	**3.98**
Overall, the TA was effective in his/her role	**4.96**	**5.00**	**4.96**	**4.14**	**4.05**

21-259 — Fall 2014 — Calculus in Three Dimensions

	Sec C	**Sec L**	**Sec N**	**Dept**	**MCS**
Response rate	53%	54%	61%	61%	57%
The TA was consistently well-prepared and organized for class	4.80	4.93	4.94	4.25	4.16
The TA made me feel free to ask questions	4.93	4.93	4.94	4.38	4.24
The TA clearly and effectively responded to students' questions and comments	4.93	4.93	4.94	4.21	4.09
The TA treated all students with respect	4.93	4.93	4.94	4.57	4.42
The TA's English was understandable	5.00	4.93	5.00	4.46	4.29
The TA was knowledgeable and familiar with concepts covered in lectures	5.00	4.93	5.00	4.43	4.30
The TA encouraged students to participate actively in recitation	4.67	4.87	4.65	4.16	4.04
The TA managed class time effectively	4.67	4.93	4.82	4.16	4.01
The TA was readily available for consultation with students	4.87	4.87	4.94	4.36	4.19
Overall, the quality of this recitation was:	**4.93**	**4.93**	**4.94**	**4.19**	**4.11**
Overall, the TA was effective in his/her role	**4.93**	**4.93**	**5.00**	**4.24**	**4.15**

21-127 — Spring 2014 — Concepts of Mathematics

	Sec A	**Sec F**	**Dept**	**MCS**
Response rate	70%	76%	61%	52%
The TA was consistently well-prepared and organized for class	4.95	4.83	4.09	4.14
The TA made me feel free to ask questions	4.95	4.96	4.18	4.21
The TA clearly and effectively responded to students' questions and comments	4.95	4.96	4.08	4.11
The TA treated all students with respect	5.00	4.92	4.38	4.39
The TA's English was understandable	5.00	4.96	4.28	4.29
The TA was knowledgeable and familiar with concepts covered in lectures	5.00	5.00	4.32	4.31
The TA encouraged students to participate actively in recitation	4.82	4.96	3.99	4.03
The TA managed class time effectively	4.77	4.96	4.03	4.03
The TA was readily available for consultation with students	4.91	5.00	4.16	4.16
Overall, the quality of this recitation was:	**4.95**	**5.00**	**4.01**	**4.04**
Overall, the TA was effective in his/her role	**4.91**	**5.00**	**4.07**	**4.12**

21-120 — Fall 2013 — Differential and Integral Calculus

	Sec A	**Sec B**	**Dept**	**MCS**
Response rate	70%	89%	66%	62%
The TA was consistently well-prepared and organized for class	4.81	4.92	4.24	4.26
The TA made me feel free to ask questions	4.71	4.88	4.28	4.13
The TA clearly and effectively responded to students' questions and comments	4.71	4.79	4.12	4.16
The TA treated all students with respect	4.76	4.83	4.46	4.51
The TA's English was understandable	4.71	4.79	4.39	4.43
The TA was knowledgeable and familiar with concepts covered in lectures	4.71	4.92	4.39	4.40
The TA encouraged students to participate actively in recitation	4.62	4.63	4.06	4.09
The TA managed class time effectively	4.62	4.83	4.12	4.14
The TA was readily available for consultation with students	4.67	4.92	4.29	4.30
Overall, the quality of this recitation was:	**4.81**	**4.96**	**4.14**	**4.17**
Overall, the TA was effective in his/her role	**4.76**	**4.96**	**4.21**	**4.25**

Section 4

Textbook

Textbook overview

An Infinite Descent into Pure Mathematics was born out of lecture notes I wrote for an introductory pure mathematics class at Carnegie Mellon University in summer 2015.

Table of contents

Design principles

- **Skills focus.** The focus of the textbook, particularly in the beginning, is on the *skills* required to pursue pure mathematics at a higher level, rather than just the knowledge. Chapter 1 *Logical structure* guides readers through the process of decomposing a statement into its logical form, and using that form to determine what proof techniques are appropriate and how to structure a proof.
- **LaTeX support.** The textbook contains a tutorial for typesetting mathematics using LaTeX. At any point when new mathematical notation is introduced, the corresponding LaTeX code is also presented. There is also an index of notation at the end of the book to assist in finding the desired LaTeX commands.
- **Exercises.** Though many proofs and examples are provided in the textbook, a large quantity of material is delivered as exercises for the reader. This is to promote problem-solving skills and inquiry from the outset. Hints to selected exercises are included in the appendix, but I made a conscious decision not to include full solutions.
- **Writing guide.** Appendix A of the textbook covers writing skills, from choosing which words and symbols to use in a clause within a sentence to structuring a substantial mathematical document.
- **Completeness.** I have tried to make sure that the material in the textbook is complete and coherent, in the sense that material later in the book depends only on material earlier in the book and that all details are included, except when the details obfuscate the intuition, in which case they are still included, but are relegated to an appendix.

Publication information

A current version of *An infinite descent into pure mathematics* can be accessed at the URL https://infinitedescent.xyz/. I am currently preparing the book for publication; the paperback will have ISBN 978-1-950215-00-3.

Use for teaching

To my knowledge, my textbook has been used as the course text for the following courses:

Institution	Course	Semester(s)/Quarter(s)	Instructor(s)
Northwestern	Math 300	Fa18 Wi19 Sp19 (Wi20)	me, Maltenfort, Richter, (+2)
Carnegie Mellon	15-151	Fa16 Fa17 Fa18 Fa19	Mackey
Carnegie Mellon	21-128	Fa16 Fa17 Fa18 Fa19	Mackey
Carnegie Mellon	21-127	Su15 Sp17 Su17 Sp18 Su18	me, Offner, Radcliffe, Allison
Johns Hopkins	Math 301	Fa19	Riehl

During semesters when other professors use my textbook, I remain in frequent contact in order to discuss the content of the book, answer their questions, and make corrections.

Student comments

Clive's notes are the bible for all mathematics students in Concepts. He rigourously explains all theorems and proofs, and accompanies them with examples such that even the layman can understand. —**Nicholas T**, *21-128 Fall 2016*.

I had never taken a math course as focussed on logic as this one, and after the first chapter I felt comfortable and confident reading fully rigorous proofs. —**Christopher W**, *21-128 Fall 2016*.

The notes were laid out in such a way that I never felt like there was a leap in logic or something that I was supposed to "just accept as part of the course." They took the reader seriously and thoroughly explained the material. —**Erik S**, *15-151 Fall 2016*.

These were really well-written notes that I think nailed the balance between explanation and asking questions without answer. I also appreciated the general wording of the notes and the detailed symbols. —**Udit R**, *21-127 Spring 2017*.

I really enjoyed trying to work out the unsolved exercises; not only was I able to read about a new theorem or definition, I could try my hand at proving a proposition afterwards.
—**Stephanie F**, *15-151 Summer 2017*.

In the opinion of someone experiencing and learning anything of such raw mathematics for the first time, this book was amazing. It provided definitions, proofs for definitions, theorems, proof for theorems, and the list just continues to grow. It was extremely useful, if not pivotal, throughout the course in enhancing my learning of the topic. —**Anonymous student**, *15-151 Summer 2017*.

Section 5

Supporting documentation and teaching materials

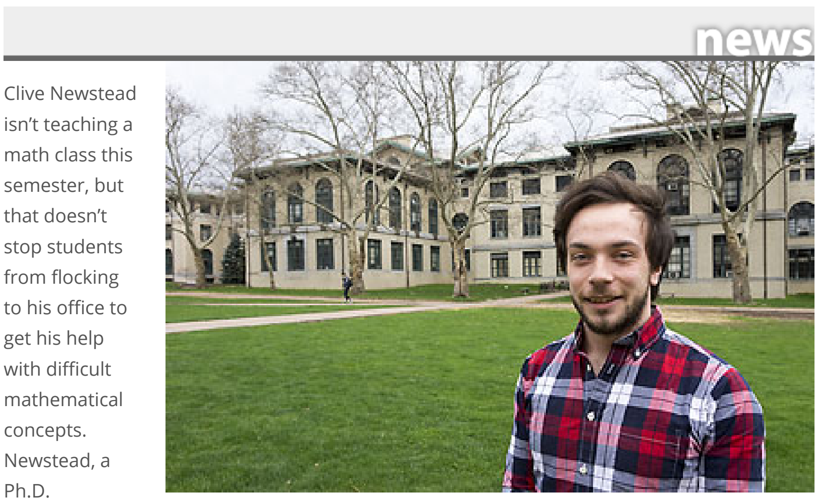
Contents

Carnegie Mellon University

Mellon College of Science

April 25, 2016

Clive Newstead Wins Carnegie Mellon's Graduate Student Teaching Award



Clive Newstead isn't teaching a math class this semester, but that doesn't stop students from flocking to his office to get his help with difficult mathematical concepts. Newstead, a Ph.D. candidate in the Department of Mathematical Sciences and this year's winner of Carnegie Mellon's Graduate Student Teaching Award, is a bit of a legend. Students often attend his recitations even though they aren't assigned to his section, and many of his former students still seek out his help long after they have had him as a TA in class.

"Clive is the type of TA that I wish I could have for every class," wrote sophomore Olivia Cannizzaro.

Newstead is known for going above and beyond to make sure his students understand complicated and often confusing mathematical concepts. He gives students careful feedback on homework problems, holds numerous office hours, is always available to answer questions by email, and hosts his own review sessions before exams. He has a natural ability to explain extremely complex ideas in a way that makes them simple to comprehend. And his passion for math has a profound impact on his students.

"His enthusiasm for and love of math reminded me each day why I came to CMU and why I want to continue studying mathematics," wrote first-year student Allison Klenk.

Newstead, who is pursuing a Ph.D. in Pure and Applied Logic, has taught several courses in the Department of Mathematical Sciences, including Mathematical Concepts and Proofs, Concepts of Mathematics, Calculus in Three Dimensions, Multivariate Analysis, and Differential and Integral Calculus. No matter the class, "he does a superb job quietly, with no bells and whistles—he makes it seem so natural and effortless,” wrote Deb Brandon, associate teaching professor of mathematical sciences and teaching assistant supervisor.

In addition to his TA duties and his own coursework and research, Newstead is exploring the science of teaching and learning. He is a Graduate Student Teaching Fellow at the Eberly Center and has enrolled in its Future Faculty Program.

“Clive will almost certainly go on to write several books, develop new courses and teach at the highest level,” wrote John Mackey, teaching professor of mathematical sciences. “He is the most unique and talented young educator of mathematics that I have had the good fortune to meet.”

4400 Fifth Avenue Pittsburgh, PA 15213

Visitors may use the Bellefield Avenue entrance.
412-268-7699

Carnegie Mellon University

Eberly Center for Teaching Excellence
and Educational Innovation
5000 Forbes Avenue
Cyert Hall 125
Pittsburgh, PA 15213

December 29, 2015

SUBJECT: Clive Newstead's completion of the Future Faculty Program

To whom it may concern:

I am writing to confirm that Clive Newstead, a Ph.D. student in Mathematical Sciences, has completed all requirements for the Future Faculty Program offered through Carnegie Mellon's Eberly Center for Teaching Excellence and Educational Innovation. The Future Faculty Program is designed to help graduate students develop and document their teaching skills in preparation for a faculty career.

This letter documents Clive's activities and describes his fulfillment of the program's four required components: (1) seminar attendance; (2) teaching feedback consultations; (3) a course and syllabus design project; and (4) an individualized project.

Requirement: Seminars

Graduate students must attend at least eight seminars through the Eberly Center, of which at least four must be designated as core seminars for future faculty. Each two-hour seminar integrates educational research and theory with practical pedagogical strategies and uses a variety of activities to draw on the experiences and reflections of the seminar participants. Clive has been an active participant in these seminars and has shared insights from his own teaching experiences and disciplinary knowledge. He has attended eleven seminars, of which five are core (indicated by an asterisk):

1) Monitoring Your Teaching Effectiveness*
2) Supporting Your Students' Academic Integrity
3) Working Well One-on-One
4) Course and Syllabus Design*
5) Motivating and Engaging Students*
6) Designing and Implementing Group Projects
7) Teaching Problem Solving
8) Building a Teaching Portfolio
9) Teaching First-Year Undergraduates
10) Providing Helpful Feedback*
11) Supporting Student Learning Through Good Assessment Practices*

Requirement: Teaching Feedback Consultations

Graduate students must receive feedback via consultations by Eberly Center consultants on at least two occasions, of which at least one must be in an authentic classroom context. Clive has received feedback on his teaching on three occasions.

In Spring 2014, Clive participated in one of the Eberly Center's Microteaching Workshops. In this workshop, each of a few graduate student participants teaches for five minutes and receives immediate feedback from the other participants and the Eberly Center consultants running the workshop. My colleague met with Clive after the workshop to watch and discuss the video of his lesson, which was about cardinality. In his lesson, he described the everyday applications of cardinality and defined concepts using both informal and formal vocabulary. He also engaged the audience by asking questions and providing positive reinforcement when audience members participated. In the follow-up conversation, Clive reflected on his lesson, and he and my colleague brainstormed a few strategies to implement in the future. For example, they discussed ways to make his board work more effective, organized, and synchronized.

In Summer 2015, my colleague observed Clive teach a class session for 21-127: Concepts of Mathematics, a course that he instructed for approximately forty students. During the observation, Clive employed several evidence-based teaching strategies. First, he targeted student motivation by making frequent connections between the course content and other disciplines: for example, he compared mathematical proofs to computer programming. Second, he used verbal cues throughout the lesson to connect concepts and to leverage the day's guiding question. Third, he modeled expert problem-solving approaches by identifying and explaining the importance of various problem-solving steps.

During the same course, Clive invited my colleague to his class to conduct an Early Course Feedback focus group in order to collect student feedback partway through the course. While Clive left the room for twenty-five minutes, my colleague asked students to list aspects of the course that were helping them learn, as well as suggestions they had to improve their learning experience. After compiling their verbal and written feedback, my colleague met with Clive to discuss the students' responses. Together, they reflected on what students thought was working well (e.g., the low-stakes practice opportunities, the example problems, and Clive's enthusiasm as an instructor) and their suggestions for improving the course (e.g., adjusting the problem sets, slowing speaking pace). During their meeting, Clive strategized about possible approaches to addressing the students' suggestions and how to communicate this with students effectively.

Requirement: Course and Syllabus Design Project

Graduate students must create a syllabus that summarizes and represents their course design decisions. For his course and syllabus design project, Clive developed a syllabus for the course 21-256: Multivariate Analysis. In his syllabus, Clive provided comprehensive information about multiple aspects of the course, from student-centered learning objectives, to descriptions of assessments, to thorough course policies. For example, the learning objectives ranged from computation and problem solving to classification and application. Clive provided an overview of the course by describing how Multivariate Analysis is split into three parts (vectors and matrices, optimization, and integration) and explaining how these parts are relevant to real-world applications. The policies section described Clive's expectations for homework submission, acceptable forms of collaboration, acceptable use of external resources, exam make-up procedures, and accommodations for students with disabilities. Finally, Clive's syllabus included the expectations he maintains for his students, what his students can expect from him, and an invitation for students to provide feedback on the course or seek help with the material as necessary – all of which contributes to a productive, positive learning environment.

Requirement: Individualized Project

Graduate students must create a set of teaching materials relevant to their teaching interests and discipline. For his individualized project, Clive developed materials for 21-127: Concepts of Mathematics, a course he taught solo in Summer 2015. For this project, Clive submitted one week's worth of (1) course notes, (2) short in-class exercises, (3) exercises associated with a longer, in-class "workshop," (4) a homework problem set, (5) a notes summary handout, and (6) a set of slides that introduced students to the final project. He also submitted a detailed written reflection that described how each of these materials were used in the course, the extent to which they were successful for student learning, and how he might revise the materials for future iterations of the course. For example, for the final project, Clive asked students to submit a substantial written piece "designed to reflect the practices of professional mathematicians." In his reflection and our conversations, Clive explained that while students found the project to be enjoyable and engaging, and while many students submitted high-quality work, some students were confused about the project and felt rushed to complete it. If he were to teach the course again, he would provide more specific expectations and instructions for the project, and he would constrain the scope of the project if the course were taught as a six-week condensed version.

Concluding thoughts

In completing the Future Faculty Program, Clive has developed significant knowledge of effective pedagogical principles, gained experience in applying these principles, and received formative feedback on his teaching. If you have any questions about the Eberly Center's Future Faculty Program, please feel free to contact me by phone (412-268-4083) or email (hdwyer@cmu.edu).

Sincerely,

Heather Dwyer, Ph.D.
Teaching Consultant
Coordinator of the Future Faculty Program

Transcript of Teaching Development Activities

Eberly Center
Teaching Excellence & Educational Innovation

Clive W Newstead Mathematical Sciences

Seminars

Each seminar integrates educational research and pedagogical strategies

Monitoring Your Teaching Effectiveness* [Sep 12, 2013]

Supporting Your Students' Academic Integrity [Sep 25, 2013]

Working Well One-on-One [Oct 03, 2013]

Course and Syllabus Design* [Oct 23, 2013]

Motivating and Engaging Students* [Jan 28, 2014]

Designing and Implementing Group Projects [Feb 12, 2014]

Teaching Problem Solving [Feb 27, 2014]

Building a Teaching Portfolio [May 23, 2014]

Teaching First-Year Undergraduates [Jun 27, 2014]

Providing Helpful Feedback* [Sep 10, 2014]

Supporting Student Learning Through Good Assessment Practices* [Mar 25, 2015]

Leveraging Diversity and Promoting Equity in Your Classroom* [Oct 12, 2016]

How much is too much? Providing undergraduates with the right level of challenge [Nov 01, 2016]

Workshops

Each workshop provides immediate feedback to participants

Microteaching Workshop [Mar 20, 2014]

2016 Teaching & Learning Summit [Oct 14, 2016]

Faculty Panel: Teaching and the Academic Hiring Process [Apr 05, 2017]

Transcript of Teaching Development Activities

Eberly Center
Teaching Excellence & Educational Innovation

Clive W Newstead Mathematical Sciences

Projects

Course & Syllabus Design Project [Dec 09, 2015]
Course: 21-256: Multivariate Analysis

Individual Project [Dec 29, 2015]

Teaching Feedback Consultations

Microteaching Video Consultation [Apr 02, 2014]

Classroom Observation [Jul 22, 2015]
Course: 21-127: Concepts of Mathematics

Early Course Feedback/Focus Group [Jul 07, 2015]
Course: 21-127: Concepts of Mathematics

Other Activities and Consultations

FFP progress consultation [Nov 23, 2015]
Met about FFP progress

FFP Overview [Apr 02, 2014]

Graduate seminar co-facilitator [Aug 26, 2015]
Revised and co-facilitated the 1-hour graduate seminar "Engaging Students in Active Learning" for science graduate students.

Graduate Teaching Fellow
Graduate Teaching Fellows work with the Eberly Center to support the professional development of CMU graduate students regarding teaching. Started January 2015.

Transcript of Teaching Development Activities

Eberly Center
Teaching Excellence & Educational Innovation

Clive W Newstead Mathematical Sciences

38-801 Evidence-Based Teaching in STEM [Dec 08, 2014]
Fall 2014, 7-unit course

Foundations of Higher Mathematics (Math 300)

Section 31, Winter 2019

Clive Newstead (instructor)
cnewstead@northwestern.edu
Locy Hall 205
(847) 467-4078

Kitty Yang (TA)
SzuYang2018@u.northwestern.edu
Locy Hall 206
(847) 467-6255

https://sites.math.northwestern.edu/~newstead/teaching/300wi19/

Section 1
Times and places

- **Class** — MoWeFr 9:00–9:50am in Lunt Hall 101

  The focus of class will be on learning material and acquiring skills (see the learning objectives below). Classes will include a mixture of lecture and activities, including problem-solving exercises, group discussions and student-led presentations.

  Class will additionally meet on Tuesday 8th January and Tuesday 19th February.

- **Discussion** — Tu 9:00–9:50am in Lunt Hall 101

  Discussion sessions will be led by Kitty. They will be less formally structured than classes, and are intended to be an opportunity for you to review the material covered in class and to strengthen your learning.

- **Office hours.** These are times when I and/or Kitty will be available to discuss pretty much anything from the course—they are a great opportunity for us to get to know you better (and vice versa). Please see the guidelines for office hours in Section 6 below.

  Office hours will be announced during class.

  If you can't attend the scheduled office hours, please contact me to schedule a meeting at another time.

- **Examinations.** There will be a midterm exam, which will be held during class, and a final exam—you might want to put the following dates in your calendar!
  - **Midterm exam.** Wednesday 13th February during class.
  - **Final exam.** Monday 18th March at 9:00–11:00am in Lunt Hall 101.

Section 2

Learning objectives

Upon successful completion of this course you should be able to:

(1) Accurately use standard mathematical notation and terminology in mathematical writing, including logical operators, quantifiers, sets and set operations, functions, equivalence relations, binomial coefficients and factorials;

(2) Write correct, clear and precise mathematical proofs, in an appropriate level of detail, of both familiar and unfamiliar mathematical results in the areas covered;

(3) Recognise and apply standard proof techniques, including direct proof, contradiction, contraposition, and weak and strong mathematical induction;

(4) Accurately recall mathematical definitions and state and prove the main theorems in the mathematical areas covered;

(5) Solve standard unseen problems in the mathematical areas covered by identifying which proof techniques and results from the course are appropriate and applying them accurately;

(6) Typeset basic mathematical documents using LaTeX, including the mathematical notation used in the course, sections, references and labels, and basic formatting.

Descriptions of some of the topics we will cover are as follows.

- **Fundamentals.** In the first few weeks we will set the scene for the rest of the course. We will learn some elementary proof techniques (including proof by contradiction and proof by induction); then we will develop a system of symbolic logic, and study the absolutely fundamental mathematical notions of *sets* and *functions*.

- **Finite and infinite sets.** We will use functions to pin down a precise notion of *size* that allows us to count the elements in a finite set. After developing some mathematical tools for counting, we will generalise this notion to study and compare the sizes of *infinite* sets.

- **Relations.** A fundamental notion in mathematics is that of a *relation*, and particularly that of an *equivalence relation*. We will study different kinds of relations, and prove that equivalence relations are 'the same thing' as *partitions*.

- **Additional topic(s).** If time permits, I will poll the class to choose a topic to cover next. Some possible options include elementary number theory, real analysis, order theory and structural induction.

- **LaTeX.** Typesetting mathematical documents is very difficult in most Office-style 'what you see is what you get' (WYSIWYG) editing software. The *de facto* standard for typesetting mathematics is LaTeX (pronunced 'lay-tek' or 'lah-tek'), in which all formatting and mathematical notation is entered as code. Many classes will include LaTeX workshops, and the course includes a LaTeX typesetting project.

Section 3

Reading materials

As course notes we will use my (freely available) book-in-progress:

- *An infinite descent into pure mathematics* by Clive Newstead
  ISBN 978-1-950215-00-3 — not yet in print
  https://infinitedescent.xyz/

The following books are also relevant and useful:

- *Book of Proof* (3rd edition) by Richard Hammack
  ISBN 978-0-9894721-2-8
  https://www.people.vcu.edu/~rhammack/BookOfProof/

- *Reading, Writing, and Proving* (2nd edition) by Ulrich Daepp & Pamela Gorkin
  ISBN 978-1-4419-9478-3

- *A Concise Introduction to Pure Mathematics* (4th edition) by Martin Liebeck
  ISBN 978-1-4987-2292-6

- *How to Prove It: A Structured Approach* (2nd edition) by Daniel J. Velleman
  ISBN 978-0-521-67599-4

A fun, non-technical discussion about how the skills you acquire when studying abstract mathematics relate to the real world is:

- *How Not to Be Wrong: The Power of Mathematical Thinking* by Jordan Ellenberg
  ISBN 978-0-14-312753-6

Section 4

Assessment and grades

Below are descriptions of the course assessments and grade assignments.

- **Classwork** (10% of total grade, lowest 3 scores dropped)**.**
  - **What?** Various short activities or exercises to be completed before or during each class, very generously graded out of a total of 5 points for each class.
  - **Why?** To keep you engaged with the material, and to provide you with the opportunity to make errors and learn from them without damaging your grade.
- **Homework** (25% of total grade)**.**
  - **What?** Several challenging mathematical exercises based on material covered in class, graded for mathematical correctness and quality of proof-writing.
  - **Why?** To give you the opportunity to show off what you've learnt, and to allow me to give you individual feedback on your progress.
- **Midterm examination** (20% or 5% of total grade—see below)**.**
  - **What?** A fifty minute exam during class time on Wednesday 13th February.
  - **Why?** To solidify the knowledge and skills you acquired during the first half of the quarter, and to serve as a milestone in the course.
- **Final examination** (35% or 50% of total grade—see below)**.**
  - **What?** A two hour exam from 9:00–11:00am on Monday 18th March.
  - **Why?** To test you on the knowledge and skills you have acquired throughout the quarter.
- **LaTeX project** (10% of total grade)**.**
  - **What?** Short write-up of a document using LaTeX, putting into practice the mathematical typesetting skills you'll learn throughout the course.
  - **Why?** To familiarise you with using LaTeX to typeset mathematical documents.

The midterm and final exams will either respectively contribute 20% and 35%, or 5% and 50%, whichever maximises your total score. This is to reward improvement through the quarter.

The preliminary grade borderlines are as follows:

A 93% **A-** 90% **B+** 87% **B** 83% **B-** 80% **C+** 77% **C** 73% **C-** 70% **D** 60%

These borderlines might be lowered, but will not be raised; for instance, a score of 83% guarantees you a grade of B or higher, even if the borderlines change.

Section 5

Tentative class schedule

The following schedule is subject to change depending on how quickly we progress through the course material. However, the dates of the midterm and final examinations are fixed.

Date			Section	Topic
Week 1	Monday	7th January	0	Getting started
	Tuesday(!)	8th January	0	Getting started
	Wednesday	9th January	1.1	Propositional logic
	Friday	11th January	1.2	Variables and quantifiers
Week 2	Monday	14th January	1.3	Logical equivalence
	Wednesday	16th January	A.3	$\LaTeX$
	Friday	18th January	1.3	Logical equivalence
Week 3	Monday	21st January	—	*No class*
	Wednesday	23rd January	2.1	Sets and set operations
	Friday	25th January	2.1	Sets and set operations
Week 4	Monday	28th January	2.2	Functions
	Wednesday	30th January	2.2	Functions
	Friday	1st February	2.3	Injections and surjections
Week 5	Monday	4th February	2.3	Injections and surjections
	Wednesday	6th February	3.1	Induction on $\mathbb{N}$
	Friday	8th February	3.1	Induction on $\mathbb{N}$
Week 6	Monday	11th February	3.1	Induction on $\mathbb{N}$
	Wednesday	13th February	—	**Midterm examination**
	Friday	15th February	A.3	$\LaTeX$
Week 7	Monday	18th February	3.2	Finite sets
	Tuesday(!)	19th February	3.3	Counting principles
	Wednesday	20th February	3.3	Counting principles
	Friday	22nd February	3.3	Counting principles
Week 8	Monday	25th February	TBA	*Topic to be determined*
	Wednesday	27th February	TBA	*Topic to be determined*
	Friday	1st March	TBA	*Topic to be determined*
Week 9	Monday	4th March	5.1	Relations
	Wednesday	6th March	5.1	Relations
	Friday	8th March	6.1	Countable sets
Week 10	Monday	11th March	6.1	Countable sets
	Wednesday	13th March	—	*Reading period*
	Friday	15th March	—	*Reading period*
	Monday	18th March	—	**Final examination 9:00–11:00am**

Section 6

Policies and guidelines

Academic honesty and integrity

My stance on academic honesty is simple: **all work you submit should accurately reflect your understanding**, and **any help you receive should be acknowledged**. This means that if someone were to ask you to explain your work, then you would be able to explain it and to say how you came to know it. What follows are some more specific descriptions of what this looks like in practice.

Collaboration. Speaking to each other about the course material and homework problems is one of the most effective ways to learn, so this is encouraged. What I ask is that you:

- **Cite your collaborators.** This means that you're giving them credit for their help, and avoids plagiarism issues. Just write a sentence at the end of your work saying who you worked with, e.g. 'I discussed Q4 with Carl Gauss, who helped me prove that f is surjective'.
- **Write your work up independently.** If you made any permanent records (such as notes or photos) during collaboration sessions, these records should be destroyed well before you write up your solution. For example, any notes on whiteboards should be erased, notes on paper should be thrown away, and photos should be deleted. Direct copying is forbidden.

External resources. Sometimes you need a little more guidance than is available from your notes, and doing some research can give you the boost you need to understand the material in the course. If you do use external resources, then please:

- **Cite your sources.** If you used a book or website, other than the course textbook or other assigned reading, please say so—just the book title and author, or web page URL, is fine.
- **Write your work up independently.** Close the book or web page and make sure you've understood what you learnt before you start writing—otherwise, all you're really doing is copying, and then your work doesn't reflect your understanding.

Examinations. Exams are the main so-called *summative assessments* of the course, meaning that they are intended to be an opportunity for you to demonstrate the knowledge and skills you have acquired. The only resource you should have available to you going into an exam is your brain—this means you should not have your notes open, you should not speak to others during the exam, and you should not be looking at other people's answers.

Instances of suspected academic integrity violations will be reported to the Weinberg Assistant Dean for Academic Integrity for further investigation. If I believe academic integrity violations are widespread, then I may resort to using plagiarism detection software.

If you are ever in doubt about whether something you are doing is in violation of academic integrity, your safest bet is to ask me as soon in advance of turning in your work as possible.

Attendance and absence

Classes will be both fun and intellectually stimulating, so you'll probably want to attend, but in case that's not enough motivation, remember that 10% of your grade comes from classwork. If you know you're going to miss a class, **please let me know as soon as possible** so that I can tell you what to do in order to catch up.

Frequent absence from class is correlated with lower academic performance, as well as other issues including mental health concerns—with this in mind, if I notice that you have developed a pattern of absence, then I will contact the Weinberg Office of Undergraduate Studies and Advising (OUSA), who will then check in with you.

If you are absent from an examination, I will work with you and your academic advisor (and/or OUSA) to work out if, how and when you can make up the work. Please let me know during the first two weeks of the quarter if you know in advance that you must miss an examination for a legitimate reason.

Homework submission

Homework is designed not just to test your knowledge, but also to help you learn. I set deadlines because it's important that you understand the content on the homework before we move on to new material in class. As such, it is in your own interests to do the homework and submit it on time.

Late homework will typically not receive credit since the grading turnaround will be quick. If there are any special circumstances that mean you absolutely need to submit the homework late, please speak to me as soon as possible, and preferably well in advance of the deadline. If in doubt, ask—the sooner you ask, the easier it will be for us to find a solution.

Contesting a grading error

Kitty and I will grade a *lot* of your work throughout the quarter, but (just like you) we are mere humans, so it is entirely possible that we will make an error from time to time. If you believe a grading error has been made, then please do the following:

1. Write a note indicating what error(s) you believe has been made—be as specific as possible, and include the relevant question number(s).
2. Attach the note to your work and hand it in at the beginning of class within three working days of the return date—for example, if homework was returned on a Friday, you should turn it in on the following Monday or Wednesday.

I will then regrade the work in question from scratch and return it to you within a week—at this point, your homework grade is final.

Extra credit

If you feel particularly enthusiastic about a topic, please speak to me—we might be able to organise some work that you can do for extra credit, such as giving a short presentation in class, writing a blog post or making a poster.

Extra credit will be reflected in your grade by increasing your overall course percentage by an agreed-upon fixed amount (usually 0.5% or 1%), up to a maximum total of 1.5% for the quarter.

Extra credit is not guaranteed. I will make every effort to provide as many opportunities as I can, but this is limited by my own resources—in particular, if you are interested, then you should ask as early in the quarter as you can. All work for extra credit must be completed before the Weinberg College Reading Period.

Make-up work

One of the best ways to learn is by making errors and then reflecting on those made. With this in mind, I will provide the opportunity to make up one question from each homework assignment by resubmitting your solution together with a written reflection exercise.

Although you will not be able to resubmit your midterm or final exam solutions for make-up credit, we will meet individually after the midterm exam to discuss how it went.

Classwork cannot be made up, but your lowest three classwork scores will be dropped. The effect of missing a small handful of classes (due to sickness, for example) is negligible—however, missing several classes will have a noticeable negative impact on your grade.

Accommodations

It is very important to me that my methods of instruction and assessment are fair to everyone. If you require any accommodations, including extra time on examinations, note-taking services, large-text format materials, alternative test locations, then you should register with AccessibleNU:

- **AccessibleNU**
  `https://www.northwestern.edu/accessiblenu/`
  (847) 467-5530 / `accessiblenu@northwestern.edu`

I am only able to make accommodations after I have received an accommodation notification from AccessibleNU—I cannot accept any other forms of evidence of need for accommodations, such as notes from doctors or emails from parents.

Mental health and wellness

It is likely that you will feel overwhelmed (with work or otherwise) at some point during the quarter. Please be aware that Northwestern University has a *lot* of resources available for helping you on your way, including:

- **Counseling and Psychological Services (CAPS)** (confidential resource)
  https://www.northwestern.edu/counseling/
  (847) 491-2151
- **Student Assistance and Support Services (SASS)**
  https://www.northwestern.edu/studentaffairs/dos/
  (847) 491-4582
- **Dean on Call** (if you don't know who else to call)
  (847) 491-8430 (Mo–Fr 8:30am–5:00pm)
  (847) 467-3022 (after hours)

More information and resources can be found on the university's Health and Wellness page:

https://www.northwestern.edu/studentaffairs/dos/resources/health-wellness/

Discrimination, harassment and sexual misconduct

We share a responsibility to make our community one where everyone has equal opportunities and feels safe, both inside and outside the classroom. I believe discrimination and harassment (sexual or otherwise) have no place in my classes or at Northwestern University, and I am committed to preventing such behaviour and taking action if it occurs. Some resources for students regarding discrimination and harrassment are as follows.

- **Center for Awareness, Response and Education (CARE)** (confidential resource)
  https://www.northwestern.edu/care/
  (847) 491-2054
- **Deputy Title IX Coordinator for Students**
  (847) 467-6571 / deputytitleixcoordinator@northwestern.edu
- **Office of Equity**
  https://www.northwestern.edu/equity/
  (847) 467-6165

Whilst I am willing to discuss sensitive issues, please bear in mind that I am required to report instances of suspected sexual misconduct to the Deputy Title IX Coordinator for Students.

Office hours

Office hours are quite simply times that Kitty and I have set aside to be available to meet with students. Make the most of them! Since office hours are entirely unstructured, please make sure that you follow the following guidelines.

- Have something in mind to discuss or ask, and be as specific as possible.
- Please take turns with other students—for example, if you ask a question, allow the other students present to ask a question before you ask a follow-up question.
- If office hours are busy, please avoid 'camping out'—my office isn't very big!
- Please don't show me your written attempts at homework solutions. (I won't look at them!) It is not fair for me to verify if your attempt is correct or to suggest how it can be improved ahead of the submission deadline.
- Please only come to the office hours for this course or 'open office hours'—if you come to my Math 290 office hours, I will prioritise questions about Math 290 (and vice versa for Math 290 students attending Math 300 office hours).

You are also very welcome to schedule individual meetings with me or Kitty—just send one of us an email.

Talk to me

I want you to learn a lot and I want you to enjoy taking this course. So that I can find out if this is happening, I encourage feedback—be it positive or negative—on all aspects of the course at any time during the quarter.

For example, if something I'm doing is making it difficult for you to learn, then say something before it's too late; or if you particularly enjoyed something we did in class, say so so that we can do it again.

You can do this by just speaking to me or Kitty, or by sending one of us an email. Giving feedback will in no way affect your grade, positively or negatively.

Rubric for final exam Q8

15 points for obtaining the correct answer using correct means

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix}$$

[Note that A is uniquely determined by the information given, so no other answers are acceptable.]

Partial credit — method #1 (using eigentools)

[2] Observing that the vectors given form an eigenbasis for A.

[5] Observing that $A = SDS^{-1}$, where $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

[4] Finding $S^{-1} = \dfrac{1}{2} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

[4] Computing $A = SDS^{-1}$.

Partial credit — method #2 (finding $A\vec{e}_i$ for $i = 1, 2, 3$)

[3] $A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$

[5] $A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{2} A \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{2} A \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

[5] $A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2} A \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \frac{1}{2} A \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$

[2] Deducing that A is as desired.

Math 300 homework grading guidelines

Clive Newstead, Winter 2019

Grading principles

- Work should be graded on its merits—no ‘negative grading’.
- Homework exercises should test students’ knowledge and skills in alignment with the course learning objectives, and the assigned numerical scores should reflect this.
- In addition to a numerical score, students should be provided with qualitative feedback on their homework solutions, aimed at helping them to improve their skills.

Homework assessment criteria

Solutions to homework exercises are graded out of 8 points against two criteria: **mathematical correctness** and **proof-writing quality**.

A score of ‘$3+2$’ indicates that the solution received a score of 5 points, achieving 3 points for mathematical correctness and 2 points for proof-writing quality.

What follows is a description of the factors affecting these criteria, and descriptions of what solutions achieving particular point levels might look like.

- **Mathematical correctness** (5 points)**.** Factors affecting this criterion include: assumptions made, accuracy and appropriateness of use of definitions and results, correctness of logical reasoning, and the degree to which the solution reaches the desired conclusion(s).

  Representative descriptions of solutions at the 5-, 3- and 1-point levels are as follows, with 0 points reserved for answers where no progress towards this criterion has been made.

  - **5 points.** Solution is correct and complete. Relevant definitions and results are applied accurately, irrelevant definitions and results are not invoked, logical reasoning is evident and valid, and the correct answer is reached.
  - **3 points.** Solution is only partially correct or partially complete. Relevant definitions and results are invoked, but may be incorrectly applied or inaccurately recalled. Logical reasoning is broadly valid, but may contain some gaps. Progress towards the correct answer is significant.
  - **1 point.** A serious attempt has been made but progress is limited. Relevant definitions or results may have been recalled, but there is not much success in their application. Logic used is questionable. Progress towards the correct answer is evident but not significant.

  Solutions may receive 4 or 2 points if they fit somewhere between two of these descriptions.

- **Proof-writing quality** (3 points)**.** Factors affecting this criterion include: clarity of writing, neatness of presentation, accuracy of use of mathematical notation and terminology, level of detail provided, and logical flow.

  Representative descriptions of answers at the 3-, 2- and 1-point levels are as follows, with 0 points reserved for answers where no progress towards this criterion has been made.

  - ◇ **3 points.** Solution is neatly and logically structured and is easy to read. Notation and terminology is used accurately, and variables are appropriately quantified. Enough detail is provided that another student in the course would feel satisfied that the solution is correct without having to ask 'why?' at any stage.
  - ◇ **2 points.** Solution is neatly presented and is easy to read overall, but contains instances where it would be difficult for another student in the course to understand, either because of a few inaccuracies in notation or terminology, or because insufficient explanation of logical reasoning is provided.
  - ◇ **1 point.** Solution is messy (but legible) or is difficult (but not impossible) to follow. Frequent errors are made with notation and terminology. Logical reasoning is rarely made explicit or is difficult to extract; another student in the course would struggle to understand the solution.

Classwork

Classwork will be graded out of 5 points:

- **2 points** will be awarded for completing the pre-class assignment.
- **1 point** will be awarded for coming to class on time.
- **2 points** will be awarded for completing work assigned during class.

Grading errors

The following steps should be taken in the event that a student believes a grading error has been made.

Step 1. The student writes a note outlining which exercise(s) they would like to be regraded and attaches it to their graded work, and then turns in the work and note to the instructor (Clive) within 3 working days of the return date.[a]

Step 2. Clive regrades the work without bias from the previous score (i.e. the score might increase, decrease or stay the same), and then returns the regraded work to the student within 5 working days of receiving it from the student.

[a]The 'return date' is the date that the graded work is handed back to students in class, not necessarily the date that the student receives their graded work—for example, if they are absent.

Math 290-1 Class 25

Wednesday 28th November 2018

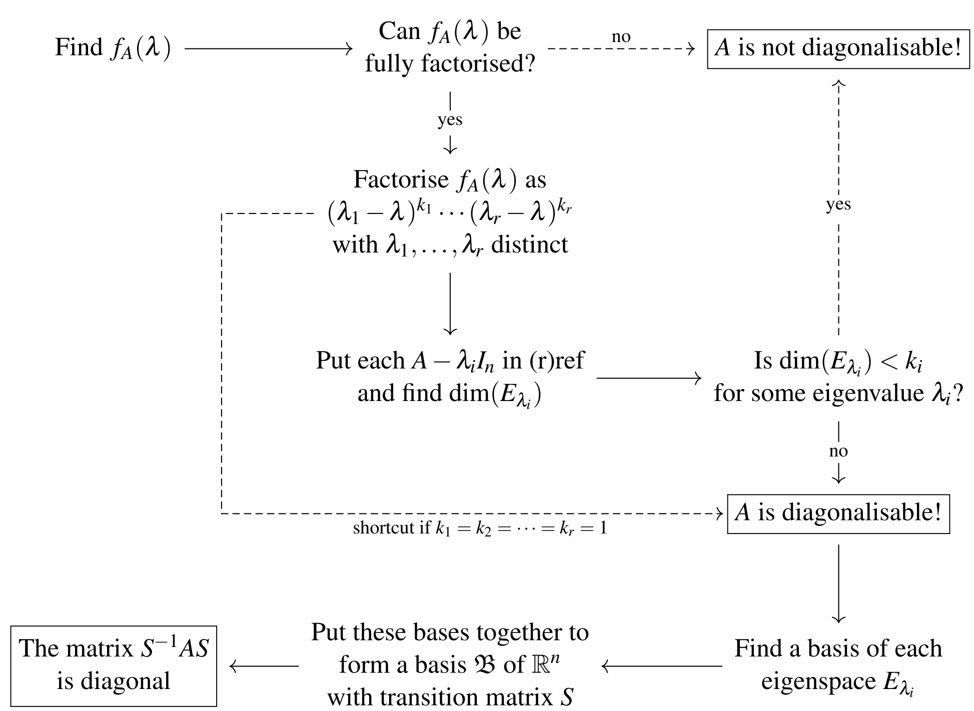
Criteria for diagonalisability

Recall that a matrix A is **diagonalisable** if it is similar to a diagonal matrix, i.e. if there is a diagonal matrix D and an invertible matrix S such that $D = S^{-1}AS$ (or equivalently $A = SDS^{-1}$).

The following conditions on a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ with $n \times n$ matrix A are equivalent:

- A is diagonalisable;
- There is a basis $\mathfrak{B}$ of $\mathbb{R}^n$ with respect to which the matrix of T is diagonal;
- There is a basis $\mathfrak{B}$ of $\mathbb{R}^n$ consisting of eigenvectors of T (called an **eigenbasis** for T);
- The sum of the geometric multiplicities of the eigenvalues of T is equal to n;
- The characteristic polynomial $f_A(\lambda)$ can be fully factorised, and the geometric multiplicity of each eigenvalue is equal to its algebraic multiplicity.

This provides us with a useful algorithm for determining if a matrix A is diagonalisable, and if it is, finding a basis with respect to which the matrix of $T(\vec{x}) = A\vec{x}$ is diagonal.



1. For each of the following matrices, determine whether or not it is diagonalisable. If it is, write down a diagonal matrix that it is similar to—you do not need to find an eigenbasis.

(a) $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

(b) $B = \begin{pmatrix} -1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

(c) $C = \begin{pmatrix} -1 & 4 & 2 \\ 1 & 2 & 2 \\ -2 & -2 & -3 \end{pmatrix}$ — you may assume that $f_C(\lambda) = -\lambda^3 - 2\lambda^2 + \lambda + 2$.

2. [*Repeated from Monday*] For each of the following statements, determine whether it is always, sometimes or never true.

(a) An $n \times n$ matrix A with n distinct eigenvalues is diagonalisable.

(b) Let A be a 3×3 diagonalisable matrix. Then $f_A(\lambda) = \lambda^3 + \lambda^2 + \lambda + 1$.

(c) Let A be a non-diagonalisable $n \times n$ matrix. Then A^2 is not diagonalisable.

3. [*Bretscher §7.1 Q72, modified*] Consider the growth of a lilac bush. At the beginning of its life the bush has one branch, and during each subsequent year of its life, each branch that already existed at the beginning of the previous year grows two new branches. (We assume that no branches ever die.)

Let $a(t)$ be the number of branches that the bush already had at the beginning of year t, and let $n(t)$ be the number of new branches that the bush grows during year t, where 'year t' is the year that *ends* when the bush is t years old.

Find closed-form expressions for $a(t)$ and $n(t)$.

15-151 class plan

Tuesday 11th July 2017

08:00 Students submit homework. Mention Alp's issue.

08:05 **Strong induction**

- ◇ Get a couple of volunteers to play *Nim*. See if students can find their way to a proof that Player 2 has a winning strategy. See why weak induction doesn't quite work.
- ◇ **Strong induction.** Compare with weak induction. Prove Player 2 has a winning strategy for Nim.
- (CW) **Exercise.** Every natural number greater than or equal to 2 can be written as a product of positive primes.

08:35 *Break*

08:40 **Recursive sequences**

- ◇ Define recursive sequence. **Example.** $a_0 = 1$ and $a_{n+1} = 2a_n$. [If ahead of time: quick proof by weak induction that $a_n = 2^n$ for all $n \in \mathbb{N}$.]
- ◇ **Example.** $a_0 = 0$, $a_1 = 1$, $a_{n+2} = 3a_{n+1} - 2a_n$ for all $n \in \mathbb{N}$. Evaluate a few terms—looks like $a_n = 2^n - 1$ for all $n \in \mathbb{N}$. Try to prove by weak induction. Remark that each step after first two relies on the previous two cases.
- ◇ Strong induction with multiple base cases—used when to get from n to $n+1$, you need to refer to a **fixed** number of previous cases. Note induction step begins with n from greatest base case. Write up proof.
- (CW) **Example.** Tribonacci sequence: $t_1 = t_2 = t_3 = 1$ and $t_{n+3} = t_{n+2} + t_{n+1} + t_n$ for all $n \geqslant 1$. Prove $t_n < 2^n$ for all $n \geqslant 1$.
- ◇ (Time permitting) **Example.** $b_0 = 1$ and $b_{n+1} = 1 + \sum_{k=0}^{n} b_k$ for all $n \in \mathbb{N}$. Prove that $b_n = 2^n$ for all $n \in \mathbb{N}$. Note it doesn't rely on *fixed* number of base cases.

09:10 *Break* — have students get their laptops ready.

09:15 **LaTeX tutorial.**

09:30 *End of class*

15-151 Homework 2

Please submit in class at 8:00am on Tuesday 11th July

Exercises

1. Read the following poorly written, albeit mostly mathematically correct, proof that $\sqrt{2}$ is irrational.

> **Proposition.** The real number $\sqrt{2}$ is irrational.
>
> *Proof.* $\sqrt{2}$ is rational, $\sqrt{2} = \frac{a}{b}$, assume cancelled to lowest terms.
>
> $$2 = \frac{a^2}{b^2}$$
> $$2b^2 = a^2 \to a^2 \text{ is even} \to a \text{ is even}$$
> $$2b^2 = (2k)^2 = 4k^2$$
> $$b^2 = 2k^2 \to b^2 \text{ is even} \to b \text{ is even}$$
>
> a and b are even, so $\frac{a}{b}$ is not cancelled to lowest terms. Contradiction! So $\sqrt{2}$ is irrational. □

   Identify at least five ways in which the proof that could be improved, and write your own version of the proof which improves on these aspects. **[10 points]**

2. Let r be a rational number, and let a and b be irrational numbers. Which of the following numbers is necessary irrational?

$$a + r \quad ar \quad a^r \quad r^a \quad a^b$$

   Prove your claims. You may assume without proof that $\sqrt{2}$ is irrational, but if you want to assume any other real number is irrational, then you must prove it. **[10 points]**

More questions on the next page

The following questions require proof by (weak) induction, which will be covered in class on Monday. If you want to get ahead, the relevant material in the book can be found in Section 1.3, up to page 58.

3. Prove by induction that
$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
for all $n \in \mathbb{N}$. **[5 points]**

4. Prove that $(1+x)^{15151} \geq 1 + 15151x$ for all $x \in \mathbb{R}$ with $x \geq -1$. **[5 points]**

5. Find and prove a formula for the following product:
$$\left(1 - \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 + \frac{(-1)^{n+1}}{n}\right)$$
where n varies over natural numbers greater than or equal to 2. **[5 points]**

Math 300 LaTeX project

Due by the end of Monday 11th March

Instructions

In this project you will typeset a mathematical document using LaTeX. Specifically, you will write a self-contained exposition of a theorem of your choice (subject to instructor approval). Your project should include:

- An introduction which motivates the theorem and provides any preliminary definitions and results needed for its statement and proof;
- A full statement and proof of the theorem;
- Examples of all concepts defined, and of applications or consequences of the theorem;
- Demonstration of your ability to incorporate the LaTeX features covered in class.

The primary assessment criteria concern the LaTeX code and resulting `.pdf` document. The basic requirements are:

- All mathematical notation and variables should be input in math mode, and all English text should be input in text mode;
- Enumeration of sections, theorems, definitions and enumerated lists must be automated using the relevant LaTeX commands and environments;
- Inline equations, displayed equations and aligned equations should be input using the respective LaTeX environments;
- References to numbered sections, results or definitions should be made using the labelling and referencing features of LaTeX;
- Your project must demonstrate the full range of features we covered in the course (see the next page for a comprehensive list);
- Your `.tex` file should compile without errors.

When complete, upload (only) the `.tex` file to the 'LaTeX project' assignment page on Canvas.

You must submit your project on Canvas by the end of Monday 11th March

You should allow enough time for technical issues to arise and be resolved, and for questions to be asked and answered. You can submit your project at any time before the deadline—the sooner the better, since your time later in the quarter would be *much* better spent preparing for the final exam!

Learning objectives

Your completed project should demonstrate your ability to:

- Use LaTeX in order to:
    - Structure a mathematical document, including using sections, subsections, paragraphs, and both bulleted and enumerated lists;
    - Format text, including using alignment, emphasis and different typefaces;
    - Typeset basic mathematical notation and equations using math mode;
    - Use theorem environments, labels and references.
- Provide a self-contained exposition of an important result, including all necessary definitions and preliminary results;
- Explain abstract concepts in a comprehensible way, including providing relevant and carefully selected examples;
- Write clear and correct mathematical definitions, result statements and proofs, which make appropriate use of variable quantification, notation and terminology.

Rubric

Your LaTeX project will be graded out of 30 points.

- **6 points** will be awarded for the quality and completeness of your exposition, including the correctness of the definitions, result statements and examples in your project, the accuracy of your mathematical language, and the clarity of your descriptions and explanations.
- **6 points** will be awarded for correctness of LaTeX code and appearance of the resulting `.pdf` document.
- **3 points** will be awarded for your project's implementation of each of the following:
    (i) **Text mode basics** — emphasis, paragraphs, alignment, and lists;
    (ii) **Math mode basics** — variables and symbols;
    (iii) **Equations** — in-line, displayed and aligned;
    (iv) **Theorem environments** — definitions, theorems and proofs;
    (v) **Document structure** — sections and subsections;
    (vi) **Labels and references**.

Project suggestions

This project is very open-ended. It can be based on anything you like, and can be as long or short as you like, provided that is based on mathematical proof and it hits all the criteria mentioned in this document.

What follows are just a few possible areas of exploration in your project. If there are other areas that interest you, please speak to Clive to determine whether that area is appropriate for your project and to develop a plan for your project.

- **Pigeonhole principle.** This is a counting argument which, when stated abstractly, says that if $m,n,q \in \mathbb{N}$ with $m > qn$, and if $f : [m] \to [n]$ is a surjection, then there is some $k \in [n]$ such that $|f^{-1}[\{k\}]| > q$. Intuitively, more than qn pigeons are distributed amongst n pigeonholes, then some pigeonhole contains more than q pigeons. This theorem has lots of interesting applications, both inside and outside of mathematics.
- **Cantor's theorem.** This theorem says that no set surjects onto its power set—that is, if X is any set, then no function $f : X \to \mathscr{P}(X)$ is surjective. Intuitively this means that $\mathscr{P}(X)$ is 'strictly larger' than X. This raises lots of questions about things like the possible sizes of infinity—for example, is there a largest infinity? Is there an uncountable set whose cardinality is less than that of $\mathscr{P}(\mathbb{N})$? How much mathematics can be done if we restrict our attention to sets whose cardinality is at most that of $\mathscr{P}(\mathbb{N})$?
- **Number theory.** The field of number theory concerns topics such as divisibility, primes and modular arithmetic. There are *lots* of theorems of number theory that would be appropriate for a LaTeX project. For example, you could prove that there are infinitely many prime numbers and explore some algorithms for listing primes; or you could prove Euler's theorem and describe how it implies the correctness of the RSA cryptosystem.

Remember to adhere to the course's and university's academic integrity policies—in particular, cite your sources and collaborators, and write up your project independently.

Math 211 blog assignment

Components

The blog assignment consists of three components:

(A) **Writing a blog post.**

Your blog post can be on any topic from calculus—some suggestions are provided later in this document. For example, your post could describe and illustrate a concept or technique from calculus, or discuss possible applications of that technique in a setting of your choice (e.g. your major area).

(B) **Discussing blog posts with classmates.**

Blogging is a social activity, so you will be expected to comment on your classmates' posts and respond to comments on your post. The are many kinds of comments that you could leave on a post—for example, you might say what you found interesting, ask a question about the topic discussed in the post, or suggest a different perspective.

(C) **Reflection.**

After you have been through the process of writing your blog post and discussing the post with your classmates in the comments section, you will reflect on the process, what you learned from it, and what you might do differently next time.

Format and timeline

Blog posts will be due on Mondays, with the first posts on Monday, May 6th, and the last posts on Monday, June 3rd; there will be at most three blog posts in a single week. You will soon receive a poll to find out your preferred posting dates—Clive will do his best to accommodate.

Each blog post will have an assigned **author** and two to three assigned **commenters**—you are allowed (in fact, *encouraged*) to comment on every blog post, but if you are listed as a commenter then you *must* read and comment on that post.

The timeline for each blog post will be approximately as follows:

- **Authors:** Meet with Clive the week before the post is due to discuss your ideas and get help; write and submit your post by the end of Monday; complete the reflection exercise after the comments on your post have been submitted.
- **Commenters:** Read the blog post and submit your comments by the end of Wednesday (except the week of the second midterm—then you have until the end of Friday).

Learning objectives and assessment

The goal of the blog assignment is to help you to develop mathematical communication skills, and to give you the freedom to explore the course content in a context that is relevant to your interests. It is supposed to be an enjoyable experience, so please take advantage of the flexibility offered by this assignment to do something that sparks your interest!

Your completed blog assignment should demonstrate your ability to:

(1) Clearly and accurately discuss concepts from calculus at a course-appropriate level;

(2) Describe a solution to a real-world problem that uses techniques from calculus;

(3) Evaluate mathematical models in terms of the extent to which they are appropriate or valid;

(4) Use LaTeX commands to typeset basic mathematical notation.

The blog assignment will be graded out of 50 points.

- **Blog post.** The post itself will be graded out of a total of 30 points, broken down as follows:
  - [10 points] **Quality of exposition.** Your blog post should be pitched at a level that is accessible to your classmates; it should demonstrate research effort; and the main take-aways from your post should be clear.
  - [10 points] **Mathematical accuracy.** Any calculations or computations that you do in your post should be correct; your mathematical notation should be accurate; and the concepts from calculus that you use should be relevant to the topic of your post.
  - [5 points] **Motivation and examples.** Your post should communicate to the reader why the subject matter is interesting, important or useful; this should be backed up with examples.
  - [5 points] **Use of LaTeX.** Any mathematical notation used in your post should be accurately typeset using LaTeX commands.

  If you submit more than one blog post, the post with the highest score will contribute to your grade.

- **Comments.** Your comments on the blog posts where you are assigned as a ‘commenter’ will be graded out of 5 points each on the extent to which you thoughtfully engage with the content of the blog post. Your comment should refer specifically to the topic or content of the post.

- **Reflection.** After the comments have been submitted to your post, you should respond to the comments and complete the reflection exercise (which will be distributed by Clive). This will be graded for completion out of 10 points.

You must submit your blog post, comments and reflection by the communicated deadlines in order to receive full credit. Plan ahead, and don’t hesitate to speak to Clive if you are having any trouble.

Blog post suggestions

Ideally you should choose your own topic based on what interests you. Some examples of blog posts that you could write include the following:

- **Tutorials.** Your blog post could be a tutorial on a concept from the course, such as differentiating certain kinds of functions (polynomial, exponential, trigonometric, . . .), using the chain rule, finding local extrema, computing definite integrals, or using the fundamental theorem of calculus. This is a great way to solidify your knowledge of a concept, or to explore a concept that you found interesting.
- **Applications.** The reason why calculus is taught to so many undergraduates is that it has widespread applications to areas outside of mathematics. In your blog post, you could discuss an example of a phenomenon in an area of your choice that can be modelled using the tools of calculus, and comment on some of the implications or limitations of such a model.
- **New topics.** Since Math 211 is a *short* course in calculus, we are not able to cover everything in full detail. If you are interested in learning and writing about topic not covered in the course, please speak to Clive for guidance.

Resources

Some resources that might help you research your blog post topic include:

- **The textbook.** There are *many* examples of concepts and applications of calculus throughout.
- **The library.** More books and research journals are available through the library (many of wihch are online)—see here:

`https://www.library.northwestern.edu/find-borrow-request/catalogs-search-tools/`

- **Your department.** If you are interested in exploring a topic relevant to your major, you could speak to a faculty member in your department to see if they can offer suggestions.

Academic integrity

This is *your* blog post, so the words, ideas and illustrations that you use should be *yours* unless you make it very clear otherwise. Remember to adhere to the course's and university's academic integrity policies—in particular, you must cite your sources and collaborators, and if you receive external help, then you should write up your blog post independently.

21-256 Exam 2: Partial Differentiation

Wednesday 18th June, 10:30–11:20am

Instructions (please read carefully)

This test is split into three sections of four questions each:

- **Section A:** Partial differentiation and the chain rule
- **Section B:** Tangent planes and linear approximation
- **Section C:** Unconstrained extrema

You should attempt **exactly ten** questions, including **at least three** questions from each section. All questions will be marked out of 10. The duration of the test is 50 minutes. Please write legibly in the blue book provided. Calculators and other electronic devices are not permitted.

Please indicate on the table below which questions you attempted.

Question	Attempted? (✓)	Score
A1		
A2		
A3		
A4		
B1		
B2		
B3		
B4		
C1		
C2		
C3		
C4		
Total	—	

Name: ____________________

Section A

A1. Find the partial derivatives of

$$f(x,y,z) = e^{x+y+z} \cos(2x + 2y + 2z + xyz)$$

with respect to x, y and z, leaving your answer in terms of x, y and z.

A2. Find a function g of variables x, y such that $\dfrac{\partial g}{\partial x} = 2xy^2$, $\dfrac{\partial g}{\partial y} = 2x^2y + 3$ and $g(0,0) = 1$.

A3. Clive is in Squirrel Hill, walking north-west in a straight line at a constant speed. He is moving north at 2 miles per hour and west at 3 miles per hour. Clive measures his happiness as being equal to his distance from campus. At the point when Clive is 2 miles due east from campus, what is the rate of change of his happiness?

A4. Let $z = f(x,y)$, where $x = s + t$ and $y = st$. Show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + x\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} + y\left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s}\frac{\partial z}{\partial t}$$

Section B

B1. Find the linearization of the function $f(x,y) = y\ln(x^2 + y^2)$ at the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$.

B2. Show that the functions $f(x,y) = \ln(1 + 2x)\cos(5y)$ and $g(x,y) = x(1-x)(2-25y^2)$ have the same linearization at the point $(0,0)$.

B3. Let Π be the tangent plane to the surface $x^2 - y^2 - z^2 = 4$ at the point $(3,-1,2)$. Find the distance from Π to the point $(6,0,0)$.

B4. Find the direction vector of the line of intersection of the tangent plane to the surface $z = x^2 + y^2$ at the point $(-2,1,5)$ and the tangent plane to the surface $xyz = 4$ at the point $(1,-2,-2)$.

Section C on next page

Section C

C1. Find the critical points of the function $h(x, y) = e^x(x^2 - 9y^2)$. For each critical point, classify it using the second derivative test or say why the test is inconclusive.

C2. Find the values of s for which the matrix $\begin{pmatrix} 2 & -1 & s \\ -1 & 2 & 1 \\ s & 1 & 2 \end{pmatrix}$ is positive definite, and verify that the function $g(x, y, z) = x^2 - xy + y^2 + yz + z^2 - 4x + 2$ attains a local minimum at $(3, 2, -1)$.

C3. The surface area of a box is $64\ \text{cm}^2$. What is its largest possible volume?

C4. Find the global maximum and minimum values of the function $f(x, y) = x^2y$ on the compact set $x^2 + 4y^2 \leqslant 3$.

Relevant formulae

You may use the following formulae:

- The distance from a point (p, q, r) to the plane passing through (a, b, c) with normal vector $\mathbf{n}$ is
$$\left| \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{a})}{\|\mathbf{n}\|} \right|$$
where $\mathbf{a} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and $\mathbf{p} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$.
- If two planes have direction vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively, then their line of intersection has direction $\mathbf{n}_1 \times \mathbf{n}_2$.

Please submit this question sheet with your answer booklet.

Full name: ______________________ **Identifier:** `57f1`

Math 300 Final Examination

Thurday 13th December 2018, 3:00–5:00pm

Please read the following instructions carefully before the examination begins.

Before the examination:

- Write your name at the top of this page, and on no other pages.
- Verify that you have been given twelve answer sheets, numbered 1 through 12, and that the identifier code at the top-right of each answer sheet is `57f1`.

During the examination:

- Answer **exactly six** questions from Section A and **exactly three** questions from Section B; write your answers on the answer sheets provided.
- Begin each new question on a new sheet of paper, and write the question number at the top of each new sheet. You may use both sides of the page.

At the end of the examination:

- Stop writing immediately when instructed to do so.
- Write the numbers of the questions you would like to be graded in the appropriate 'Question' column on the score sheet (on the opposite side of this page). Note that any additional answers will not be graded for credit, so choose wisely!
- Turn this sheet in together with **all twelve** answer sheets, even those you have not used and those you do not wish to have graded.

Additional remarks:

- If you have a question, please raise your hand and wait for a proctor to come to you.
- Work that is crossed out, illegible or not written on an answer sheet will not receive credit.
- You may not use external resources at any point during the examination—this includes lecture notes, books, phones, laptops, calculators, and your nearby classmates' answers.

Identifier: `57f1`

Score sheet

Please write the numbers of the questions you would like to be graded in the appropriate 'Question' column below. Your scores on those questions will be indicated in the table when they have been graded—please see the comments on your written solutions for more detailed feedback.

Section A		
Question	**Score**	**Points**
		8
		8
		8
		8
		8
		8

Section B		
Question	**Score**	**Points**
		24
		24
		24

Total	out of 120

Section A

Answer **exactly six** questions in this section. Each question is worth 8 points.

1. Define functions $f_n : \mathbb{R} \to \mathbb{R}$ for all $n \in \mathbb{N}$ by letting
$$f_0(x) = x - 1 \quad \text{and} \quad f_{n+1}(x) = (x^{2^n} + 1) f_n(x)$$
for all $x \in \mathbb{R}$ and all $n \in \mathbb{N}$. Prove by induction that $f_n(x) = x^{2^n} - 1$ for all $x \in \mathbb{R}$ and all $n \in \mathbb{N}$.

2. Given sets X and Y, define their *intersection* $X \cap Y$ and their *union* $X \cup Y$, and prove that $X \cap Y = X \cup Y$ if and only if $X = Y$.

3. Define what it means for a function $f : X \to Y$ to be *injective*, and prove that a function $f : X \to Y$ is injective if and only if $|f^{-1}[\{y\}]| \leqslant 1$ for each $y \in Y$.

4. Let $k, n \in \mathbb{N}$. Define the *binomial coefficient* $\binom{n}{k}$, and prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$ for all $k, n \in \mathbb{N}$ with $n, k \geqslant 1$.

5. Define what it means for a set X to be *countably infinite*, and use this definition prove that the set $\{1, 3, 9, 27, 81, \dots\}$ of all powers of 3 is countably infinite.

6. Let A and B be events in a discrete probability space $(\Omega, \mathbb{P})$ with $\mathbb{P}(A) > 0$ and $0 < \mathbb{P}(B) < 1$. Define the *conditional probability* $\mathbb{P}(A \mid B)$ of A given B, and use your definition to prove that
$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(A \mid B)\mathbb{P}(B)}{\mathbb{P}(A \mid B)\mathbb{P}(B) + \mathbb{P}(A \mid B^{\mathsf{c}})\mathbb{P}(B^{\mathsf{c}})}$$
[You may assume without proof that $A = (A \cap B) \cup (A \cap B^{\mathsf{c}})$.]

7. Let $(\Omega, \mathbb{P})$ be a discrete probability space. Define what it means for N to be a $\mathbb{Z}$-*valued random variable* on $(\Omega, \mathbb{P})$, and prove that, for all $m, n \in \mathbb{Z}$, if $\mathbb{P}(\{N = m\} \cap \{N = n\}) > 0$, then $m = n$.

8. Define what it means for a relation $\sim$ on a set X to be an *equivalence relation*, and prove that there is an equivalence relation $\sim$ on $\mathbb{R}$ defined for all $x, y \in \mathbb{R}$ by $x \sim y$ if and only if $x = y + n$ for some integer n.

Section B

Answer **exactly three** questions in this section. Each question is worth 24 points.

9. A set X satisfies the *ascending chain condition* if for every sequence $U_0, U_1, U_2, \dots$ of subsets of X, if $U_n \subseteq U_{n+1}$ for all $n \in \mathbb{N}$, then there is some $N \in \mathbb{N}$ such that $U_n = U_N$ for all $n \geqslant N$. Prove that a set is finite if and only if it satisfies the ascending chain condition.

10. Fix $n \in \mathbb{N}$. Define $U \oplus k$ for all $U \subseteq [n]$ and all $k \in [n]$ as follows.

$$U \oplus k = \begin{cases} U \setminus \{k\} & \text{if } k \in U \\ U \cup \{k\} & \text{if } k \notin U \end{cases}$$

Let $P^+ = \{U \subseteq [n] \mid |U| \text{ is even}\}$ and $P^- = \{U \subseteq [n] \mid |U| \text{ is odd}\}$.

(a) Prove that there is a bijection $f : P^+ \to P^-$ defined by $f(U) = U \oplus n$ for all $U \in P^+$.

(b) Use part (a) to deduce that $\displaystyle\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$.

11. Prove by double counting that $\displaystyle\sum_{k=r+1}^{n-r} \binom{k-1}{r}\binom{n-k}{r} = \binom{n}{2r+1}$ for all $n, r \in \mathbb{N}$.

12. Let $\sim$ be an equivalence relation on an uncountable set X.

(a) Prove that if each equivalence class of $\sim$ is countable, then $X/{\sim}$ is uncountable.

(b) Give an example of an uncountable set X and an equivalence relation $\sim$ on X, such that $X/{\sim}$ is uncountable and such that each equivalence class of $\sim$ is also uncountable.

13. Let Ω be a countable set and let $f : \Omega \to \mathbb{R}$ be a function, such that $f(\omega) \geqslant 0$ for all $\omega \in \Omega$ and $0 < \sum_{\omega \in \Omega} f(\omega) < \infty$.

(a) Prove that there exist a unique constant $k \in \mathbb{R}$ and a unique probability measure $\mathbb{P}_f$ on Ω such that $\mathbb{P}_f(\{\omega\}) = kf(\omega)$ for each $\omega \in \Omega$.

(b) Let $(\Omega, \mathbb{P})$ be a probability space and let $B \subseteq \Omega$ be an event with $\mathbb{P}(B) > 0$. Find a function $f : \Omega \to \mathbb{R}$ such that $\mathbb{P}_f(A) = \mathbb{P}(A \mid B)$ for each $A \subseteq \Omega$, where $\mathbb{P}_f$ is the probability measure on Ω determined by f as in part (a).

14. Let R and S be binary relations on a set X. The *composite* of R and S is the relation $S \circ R$ defined for all $x, y \in X$ by

$$x\,(S \circ R)\,y \quad \Leftrightarrow \quad x\,R\,z \text{ and } z\,S\,y \text{ for some } z \in X$$

(a) Prove that a relation R on a set X is transitive if and only if $\mathrm{Gr}(R \circ R) \subseteq \mathrm{Gr}(R)$.

(b) Given a function $f : X \to X$, define a relation R_f on X by

$$x\,R_f\,y \quad \Leftrightarrow \quad f(x) = y$$

for all $x, y \in X$. Prove that if $f, g : X \to X$ are functions, then $R_g \circ R_f = R_{g \circ f}$.

End of portfolio