15-151 LaTeX project

Due at 5:00pm on Sunday 16th July

Instructions

Attached is a hand-written mathematical document. You should use the LaTeX skills that you have developed over the course of the week (Monday 10th to Friday 14th July) to create a typeset version of this document, with the name and date at the top replaced with your own name and the date of submission.

Your typeset document should use the LaTeX features corresponding to the features of the hand-written document. In particular:

- All mathematical symbols and variables should be input in math mode, and all English text should be input in text mode;
- Enumeration of sections, theorems, definitions and enumerated lists must be done using the relevant LaTeX commands and environments;
- In-line equations, displayed equations and aligned equations should be input using the respective LaTeX environments;
- References to numbered sections, results or definitions should be made using the labelling and referencing features of LaTeX.

Your `.tex` file should compile without errors.

When complete, download the `.tex` file of your project and rename it to `YourAndrewID-latex.tex`; for example, Clive's would be `cnewstea-latex.tex`. Upload this file to the LaTeX project page on Canvas.

You must submit your project by 5:00pm on Sunday 16th July

When you have uploaded your project, compile and print the resulting `.pdf` file and hand it in at the beginning of class (8:00am) on Monday 17th July.

Try not to procrastinate! Allow enough time for technical issues to arise and be resolved, and for questions to be asked and answered.

Learning objectives

Upon successful completion, you should be able to perform the following tasks in LaTeX:

- Handle basic aspects of document structure, including sections and subsections, paragraphs, and bulleted and enumerated lists;
- Format text, including alignment, emphasis and fonts;
- Typeset basic mathematical notation and equations;
- Use theorem environments, referencing and labels to structure a mathematical document.

It is against these learning objectives that your project will be assessed.

Rubric

The LaTeX project is assessed against the following five criteria, each of which is worth 10 points, giving a total of 50 possible points.

- **Accuracy and appearance:** the extent to which the typeset document resembles the original, accurately reflects its content, and is readable.
- **Document structure:** use of sections and subsections, paragraphs, and both bulleted and enumerated lists.
- **Text formatting:** use of emphasis (e.g. bold face, italic or underlined text), text alignment and different fonts.
- **Mathematical notation.** use of math mode to typeset mathematical notation, appropriate use of variables and symbols, in-line and displayed equations, and aligned equations.
- **Results, definitions and references:** appropriate use of definition, theorem and proof environments, and use of labels and references.

Both the `.tex` file and the printed `.pdf` document will be assessed.

Do not worry about line-breaks or page-breaks being in the same place. Also, emphasised text in the original is <u>underlined</u>, but you may use *italic* or **bold face** text for emphasis instead if you so wish.

Prime numbers

Carl F. Gauss, Wednesday 30th April 1777

1. Introduction

Recall that a positive integer is prime if it has exactly two positive divisors. Equivalently:

Definition 1 Let $n \in \mathbb{Z}$ with $n > 0$. Then n is prime if there do not exist $a, b \in \mathbb{Z}$ such that $1 < a < n$ and $1 < b < n$ and $ab = n$.

A natural question to ask is:

How many primes are there?

There are certainly at least five: 2, 3, 5, 7 and 11 are all prime.

Theorem 2 Let $n \in \mathbb{Z}$ with $n > 1$. Then n can be factored uniquely as:

$$n = p_1^{k_1} \times p_2^{k_2} \times \cdots \times p_r^{k_r}$$

where:

(i) $p_1, p_2, \ldots, p_r$ are all prime;

(ii) $p_1 < p_2 < \cdots < p_r$;

(iii) $k_1, k_2, \ldots, k_r$ are all positive natural numbers.

Proof We prove existence and uniqueness separately:

- Existence. Left as an exercise to the reader.
- Uniqueness. Left as an exercise to the reader.

So we're done! $\square$

Proposition 3 Let $m \in \mathbb{N}$ and suppose that $m > 2$ and m is even. Then $2^m - 1$ is not prime.

Proof Since m is even, we have $m = 2k$ for some $k \in \mathbb{Z}$. Moreover, $k > 1$ since $m > 2$. Now

$$\begin{aligned} 2^m - 1 &= 2^{2k} - 1 && \text{since } m = 2k \\ &= (2^k)^2 - 1^2 && \text{by laws of exponents} \\ &= (2^k - 1)(2^k + 1) && \text{difference of two squares} \end{aligned}$$

Moreover:

- $2^k - 1 > 2^1 - 1 = 1$, since $k > 1$;
- $2^k - 1 < 2^k < 2^{2k} = 2^m$;
- $2^k + 1 > 1$, since $2^k > 0$; and
- $2^k + 1 < 2^m$, since:

$$2^k + 1 < 2^k + 2^k = 2^{k+1} < 2^{k+k} = 2^{2k} = 2^m$$

By Definition 1, we see that $2^m - 1$ is not prime. $\square$

2. Infinitude of primes

In this section, we prove there are infinitely many primes.

Theorem 4 There are infinitely many primes.

Proof This follows from the fundamental theorem of arithmetic (Theorem 2). The details are omitted. $\square$