

Fun problems

1. Alex has 18 oaks in a garden, each having the same number of acorns. After the wind some acorns felt down: from some trees exactly half felt down, from some trees exactly a third, and from the rest no acorns felt down. In total exactly a ninth of all acorns felt down. How many trees did not lose acorns at all?
2. There are a lot of wild monkeys in Brazil. Every year in January second the monkeys are recounted. In 1999 the number of monkey increased by exactly 5% compared to 1998. In 2000-2003 the increase also was exactly 5% compared to previous year. In 2003 the number of monkeys by the report was less than 5 millions. How many monkeys lived in Brazil in 2003?
3. Is it possible to fill in a 3×3 table with distinct 4-digit integers so that the sum in every two neighboring cells is divisible by 2003?
4. Positive integer n has two distinct divisors a and b such that $(a - 1)(b + 1) = n - 2$. Prove that $2n$ is a perfect square.
5. Is it possible to color some of the cells in a 10×10 square so that every cell has exactly two colored neighbors?
6. You are given 2003 integers that sum up to zero. You are allowed to choose any 300 numbers and either change the sign of all of them, or reduce each number by 1. Prove that using this operations you can transform given numbers to all zeros.
7. In ARML-land there are a lot of students (definitely more than 100). A student is called active if they know at least a 100 other students. Prove that there are either two active student who know each other or two inactive student who don't know each other.
8. A 300×300 square is cut into a 1×3 rectangles. In every vertical rectangle the column number is written. Prove that the sum of all written numbers is divisible by 3.