Similarity

Varsity Practice 1/12/20

Da Qi Chen

1 Warm-Up Problems

1. Let M be the point of intersection of the three altitudes of a triangle ABC. If AB=CM, then determine the angle of ACB.

2. In triangle ABC, altitude BE is extended to G such that EG has the same length as the altitude CF (of triangle ABC). Extend BA to H such that GH is parallel to CA. Prove that $AH = AC$.

3. Let $ABCD$ be a trapezoid where the long base AB is 90 unit long. Let E, F be the midpoints of the diagonals and suppose EF is 4 unit long. What is the measurement of the shorter base?

4. Let M be the midpoint of a base CD of the trapezoid $ABCD$. Let F be the point of intersection of AM and the diagonal BD. Extend a line parallel to CD at F so it intersects AD, AC and BC at E, G, H respectively. Prove that $EF = FG = GH$.

5. Let A, K, L, B be four sequential points on a line such that $(AL)^2 = (AK)(AB)$. Let P be another point such that $AP = AL$. Prove that PL bisects $\angle KPB$. Also, prove the converse.

2 Problem Set

1. (2003 AMC 10A 22) In rectangle $ABCD$, let $AB = 8, BC = 9, H$ is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects AH at G, and F is on AD such that $GF \perp AF$. Find the length of GF.

2. Point O is the center of the circle inscribed in triangle ABC. Let D, E be on AB, BC respectively such that $AD \cdot AC = (AO)^2, CE \cdot CA = (CO)^2$. Prove that DOE is collinear.

3. Two flagpoles of length $40, 60$ meters are placed near each other. Two anchoring lines are draw from the tip of the poles to the base of the other pole. How high is the intersecting point of the two lines?

4. Let $ABCD$ be a parallelogram. Let E, F be points on the diagonal AC such that $AE = FC$ (where E is closer to A than to C). Extend BE and BF such that they intersect AD and CD and H and G respectively. Prove that HG is parallel to AC. If we assume E is closer to C instead and assume the points H, G lies on the extension of AD, CD respectively (so they are outside of the parallelogram ACD), is HG still parallel to AC? What if E, F are on the exterior of the parallelogram?

5. Let AC be the long diagonal of a parallelogram $ABCD$. Perpendicular lines CE, CF are dropped from C to the extension of the lines AB, AD respectively. Prove that $AB \cdot AE + AD \cdot AF = (AC)^2$.

6. In triangle ABC, let E be a point on AC, O be the midpoint of BE. Extend the lines AO and CO such that it intersects the lines BC and AB at D and F respectively. If $FO = 4, OC = 15, AO = 12$ find OD and the fraction AE/EC.

7. In triangle ABC, let D be a point on AB such that $AD/DB = 2$. Let E be a point such that $BE/EC = 4$. Find the fraction CF/FD.

8. In triangle ABC, suppose $\angle BAC = 120$. Let AD be the angular bisector at A. Express the length of AD in terms of AB and AC.

9. In a trapezoid $ABCD$, let E be the intersection of the two diagonals. Let FG be a line parallel to the bases passing through E where F, G are on the sides of the parallelogram. Prove that FG is the harmonic mean of the two bases.

10. Given parallelogram $ABCD$, let E be a point on BC. Let F, G be the point of intersection of the extended line of AE and the extended line of CD and BD respectively. Prove that AG is one half the harmonic mean of AE and AF. If E is on the extension of BC, prove the same result holds.

11. In triangle ABC, let AM be a median to the side BC and P be a point on AM. Extend BP, CP to D, E on sides AC, AB respectively. Prove that DE is parallel to BC. (Hint: $GCPB$ margolellarap a etaerc)

12. Let P be any point on the altitude CD of a triangle ABC such that P is inside the triangle. Extend AP, BP to points Q, R on line BC, AC respectively. Prove that CD bisects $\angle RDQ$. (Hint: R, Q ta DC ot lellarap senil ward)