Fermat’s Little Theorem Practice

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Problems

1. Find $3^{31} \mod 7$.
2. Find $2^{35} \mod 7$.
3. Find $128^{129} \mod 17$.
4. (1972 AHSME 31) The number $2^{1000}$ is divided by 13. What is the remainder?
5. Find $29^{25} \mod 11$.
6. Find $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60} \mod 7$.
7. Let $a_1 = 4$, $a_n = 4^{a_{n-1}}$, $n > 1$
   Find $a_{100} \mod 7$.
8. Solve the congruence $x^{103} \equiv 4 \mod 11$.
9. Find all integers $x$ such that $x^{86} \equiv 6 \mod 29$.
10. What are the possible periods of the sequence $x, x^2, x^3, \ldots$ in mod 13 for different values of $x$?
    Find values of $x$ that achieve these periods.
11. If a googolplex is $10^{10^{100}}$, what day of the week will it be a googolplex days from now? (Today is Sunday)
12. Suppose that $p$ and $q$ are distinct primes, $a^p \equiv a \pmod q$, and $a^q \equiv a \pmod p$. 
    Prove that $a^{pq} \equiv a \pmod {pq}$.
13. Find all positive integers $x$ such that $2^{2x+1} + 2$ is divisible by 17.
14. An alternative proof of Fermat’s Little Theorem, in two steps:
   (a) Show that $(x+1)^p \equiv x^p + 1 \pmod p$ for every integer $x$, by showing that the coefficient of $x^k$ is the same on both sides for every $k = 0, \ldots, p$.
   (b) Show that $x^p \equiv x \pmod p$ by induction over $x$.
15. Let $p$ be an odd prime. Expand $(x-y)^{p-1}$, reducing the coefficients mod $p$. 