Combinatorial Optimization

Problem set 1

Assigned Thursday, May 28, 2015. Due Monday, June 1, 2015.

1. Formulate a linear program for the following optimization problem. Then solve your linear program with Maple and interpret the results. (Please include a printout of your Maple worksheet.)

> The Ace Refining Company produces two types of unleaded gasoline, regular and premium, which it sells to its chain of service stations for \$36 and \$42 per barrel, respectively. Both types are blended from Ace's inventories of refined domestic and refined foreign oil and must meet the following specifications:

	Maximum vapor pressure	Minimum octane rating	Maximum demand, bbl/wk	Minimum deliveries, bbl/wk
Regular	23	88	100,000	50,000
Premium	23	93	20,000	5,000

> The characteristics of the refined oils in inventory are:

	Vapor pressure	Octane rating	Inventory, bbl	Cost, \$/bbl
Domestic	25	87	40,000	16
Foreign	15	98	60,000	30

> What quantities of the two oils should Ace blend into the two gasolines in order to maximize weekly profit? (Assume that vapor pressure and octane ratings combine linearly in a blend.)

2. Formulate a linear program for the following optimization problem. Carefully draw the feasible region (i.e., the set of feasible solutions). Solve the linear program graphically. Draw at least three level curves of the objective function, including the level curve corresponding to the optimal value. (To get a nice, careful drawing of the feasible region and level curves, I suggest you use graphing software, or at least graph paper and a ruler.)

> A kennel owner has a choice of two dog foods to buy in bulk quantities to feed the dogs under her care. The average dog in her kennel needs at least 36 grams of protein, 12 grams of fat, and 9 milligrams of iron a day. An ounce of Dog Grub includes 3 grams of protein, 1.5 grams of fat, and 0.5 milligrams of iron. An ounce of Canine Chow includes 2 grams of protein, 0.5 grams of fat, and 0.75 milligrams of iron. If Dog Grub and Canine Chow cost \$0.12 and \$0.15 per ounce, respectively, how much of each should she buy to meet the daily needs of the average dog at a minimum cost?

3. Give a (simple) example of each of the following:

(a) An infeasible linear program.
(b) An unbounded linear program (i.e., a linear program with an unbounded objective value).
(c) A linear program with a unique optimal solution.
(d) A linear program with a nonunique optimal solution.
(e) A linear program with an unbounded feasible region that does *not* have an unbounded objective value.

4. A *convex combination* of two vectors x and x' (of the same size) is a vector of the form $\lambda x + (1-\lambda)x'$, where λ is a scalar in the interval $[0, 1]$. Let $A = [a_{ij}]$ be an $m \times n$ matrix and let $b = [b_1, \ldots, b_m]^{\mathrm{T}}$ be an $m \times 1$ column vector. Consider the set of constraints $Ax \leq b$, where x is an $n \times 1$ column vector. (The inequality $Ax \leq b$ means that each coordinate of Ax is less than or equal to the corresponding coordinate of b; so it is a set of m inequalities.) Suppose that $x = [x_1, \ldots, x_n]^{\mathrm{T}}$ and $x' = [x'_1, \ldots, x'_n]^{\mathrm{T}}$ are both feasible solutions to this set of constraints. Prove that any convex combination of x and x' is also feasible.

5. Here is another example of a combinatorial optimization problem. We have not yet discussed methods for solving problems like this, but see if you can find a good approach. Your goal is to find the optimal solution to the problem and to *prove* that it is optimal.

> Indiana Jones has made it through the deadly traps of an ancient temple and has discovered ten treasures inside. Unfortunately, his knapsack is too small to carry them all, so he must choose (wisely). He has made the following estimates of the objects' weights and values. If his knapsack can hold at most 20 pounds of treasure, which objects should he take to maximize the value of his loot?

Treasure	Weight	Value
Crown of Atahualpa	4 lb.	$ 4,000
Itzcoatl's Orb	6 lb.	9,000
Tablet of the Heavens	7 lb.	7,000
Golden Quetzal	13 lb.	17,000
Mask of the Ancients	5 lb.	5,000

Treasure	Weight	Value
Key of Silver Light	2 lb.	$ 2,000
Idol of Inti	10 lb.	15,000
Eternal Quipu	5 lb.	8,000
Goblet of Uxmal	3 lb.	4,000
Sacred Stone of Cuzco	8 lb.	11,000

6. The phrase "combinatorial optimization" is a mouthful. Come up with a shorter way to say it.