ILLUMINATING A ROOM

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INTRODUCTION

"Illuminating a room" is a very interesting topic as it uses a variety of applications of calculus. This project is basically about finding the height at which the light source should be placed to get the maximum amount of illumination. The height at which light source should be placed to get maximum amount of illumination is

PART A

1. OBJECTIVE

We would like to maximize the amount of illumination by adjusting the height at which the light source should be placed.

2. IDENTIFYING THE CONSTANTS

We know that:

- Intensity of light source (I)
- Source of light source (S)
- A
- B
- C
- D
- θ
- L

3. REDUCING THE OBJECT FUNCTION TO ONE VARIABLE

Now we have sin θ and d in terms of only one variable that is x.

\[
\sin \theta = \frac{L}{\sqrt{x^2 + 169}}
\]

Using the following formulas, we can differentiate this function.

\[
\text{Quotient rule: } \frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}
\]

Finally, we get:

\[
\frac{d}{dx} \left( \frac{L}{\sqrt{x^2 + 169}} \right) = 0
\]

4. DOMAIN OF THE OBJECTIVE FUNCTION

We know that "x" in the objective function represents height, so it cannot be negative. We also know that the maximum height at which the light source can be placed cannot be more than 10 feet from the ground. Therefore, x = 0 to 10.

The domain of the function is x ∈ [0, 10].

5. DIFFERENTIATING THE OBJECTIVE FUNCTION

We must now differentiate the objective function in order to find the critical numbers.

Our objective function is:

\[
L = \frac{kx}{(x^2 + 169)^{3/2}}
\]

To differentiate this function, we use three different rules of differentiation.

- Quotient rule: \[\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}\]
- Chain rule: \[\frac{d}{dx} (u(v(x))) = u'(v(x)) \cdot v'(x)\]
- Constant multiple rule: \[\frac{d}{dx} (cu) = c \frac{d}{dx} (u)\] where c is a constant

After simplifying, we get:

\[
\frac{d}{dx} \left( \frac{kx}{(x^2 + 169)^{3/2}} \right) = 0
\]

5.1) Differentiating L1, we get:

\[
L_1 = \frac{k(3x^2 + 169)}{(x^2 + 169)^{5/2}}
\]

5.2) Differentiating L2, we get:

\[
L_2 = \frac{-3kx}{(x^2 + 169)^{3/2}}
\]

5.3) Differentiating L3, we get:

\[
L_3 = \frac{k}{x^2 + 169}
\]

Therefore, we have only one critical number that is \(x = \frac{13}{\sqrt{2}}\)

6. FINDING THE CRITICAL NUMBERS

Now we must find the values of L by plugging in the critical numbers.

\[
L(x) = \left\{ \begin{array}{ll}
0 & \text{if } x = 0 \\
0.00227 & \text{if } x = \left( \frac{13}{\sqrt{2}} \right) \\
0.00226 & \text{if } x = 10
\end{array} \right.
\]

7. TESTING THE CRITICAL NUMBERS

We must now find the value of L by plugging in the three values of x that is 0, \(\left( \frac{13}{\sqrt{2}} \right)\), and 10.

<table>
<thead>
<tr>
<th>x</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\left( \frac{13}{\sqrt{2}} \right)</td>
<td>0.00227</td>
</tr>
<tr>
<td>10</td>
<td>0.00226</td>
</tr>
</tbody>
</table>

8. ANSWERING THE QUESTION

(a) We would like to maximize the amount of illumination by adjusting the height at which the light source should be placed.

(b) Given: Amount of illumination (L) = \(\frac{kx}{(x^2 + 169)^{3/2}}\)

(c) Differentiating the objective function, we get:

\[
\frac{d}{dx} \left( \frac{kx}{(x^2 + 169)^{3/2}} \right) = 0
\]

Therefore, the value of x is \(\frac{13}{\sqrt{2}}\) feet.

II. PART (B)

1. OBJECTIVE

We want to maximize the illumination by adjusting the height at which the light source should be placed.

2. IDENTIFYING THE CONSTANTS

We know that:

- A
- B
- C
- D
- Θ
- L

3. REDUCING THE OBJECT FUNCTION TO ONE VARIABLE

Now we have Sin Θ and d in terms of only one variable that is x.

\[
\text{Sin } \Theta = \frac{L}{\sqrt{x^2 + 169}}
\]

4. DOMAIN OF THE OBJECTIVE FUNCTION

We know that "x" in the objective function represents height, so it cannot be negative. We also know that the maximum height at which the light source can be placed cannot be more than 10 feet from the ground.

Therefore, 0 ≤ x ≤ 10.

The domain of the function is x ∈ [0, 10].

5. DIFFERENTIATING THE OBJECTIVE FUNCTION

We must now differentiate the objective function in order to find the critical numbers.

Our objective function is:

\[
L = \frac{kx}{(x^2 + 169)^{3/2}}
\]

To differentiate this function, we use three different rules of differentiation.

- Quotient rule: \[\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}\]
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After simplifying, we get:

\[
\frac{d}{dx} \left( \frac{kx}{(x^2 + 169)^{3/2}} \right) = 0
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L_1 = \frac{k(3x^2 + 169)}{(x^2 + 169)^{5/2}}
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L_2 = \frac{-3kx}{(x^2 + 169)^{3/2}}
\]

5.3) Differentiating L3, we get:

\[
L_3 = \frac{k}{x^2 + 169}
\]

Therefore, we have only one critical number that is \(x = \frac{13}{\sqrt{2}}\)

6. FINDING THE CRITICAL NUMBERS

Now we must equate \( \frac{dL}{dx} = 0 \) to zero or check if the derivative is undefined at any value of x.

\[
\frac{dL}{dx} = \frac{3kx(x^2 + 169) - k(3x^2 + 169)}{((x^2 + 169)^{5/2})}
\]

Finally, we get:

\[
\frac{dL}{dx} = 0 \Rightarrow (x^2 + 169)^{3/2} = 0
\]

Therefore, \(x = \frac{13}{\sqrt{2}}\)

7. TESTING THE CRITICAL NUMBERS

We must now find the value of L by plugging in the critical number that is \(x = \frac{13}{\sqrt{2}}\).

\[
L \approx 0.00549
\]

8. ANSWERING THE QUESTION

We observe that this arrangement changes the height (L = 0.00549 ft) at which the lights should be hung to best illuminate the corners of the floor.