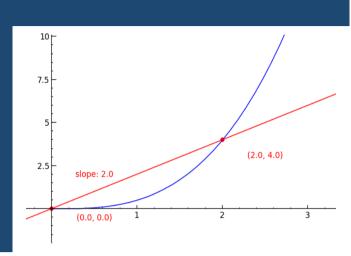
Bird Migration: Sally, Farah, Ola

We will present a problem that involves derivatives and apply these methods to solve it. Derivatives are the slope of tangent lines. Usually, when we try to find the slope of a line, we need 2 points. However, tangent lines intersect the graph in only one point, and this is when derivatives come to play their role.



Objective:

Minimize the power

Constraint:

Velocity

Domain:

Velocity is a positive number.

Formulas we are familiar with:

- 1) P=F*v
- 2) P=E/t
- 3) V=d/t

Given:

 $P(v) = (17v^{-1}) + 10^{-3} * v^{3}$ J/s

Energy= 5*10^(4) J

therefore,

 $t=P/E=(5*10^4)/(17v^-1 + 10^3*v^3)$ $v=d/t= (5*10^4)v/(17v^-1+10^-3*v^3)$

a)The question asks about the velocity that minimizes power consumption.

We need to express all variables in terms of v

Steps to solve part a:

1- Differentiate:

 $P'(v) = (-17v^{2}) + (3v^{2}/1000)$

2- find critical points:

#When P'(v)=0

 $V=(17000/3)^1/4 = 8.676 \text{ m/s}$

#When P'(v) does not exist:

None.

3-check if the critical point is a minimum using the First Derivative Test:

By picking a number less than v (v=7) and a number greater than v (v=9) ,then substituting them in the formula P'(v)

P'(7) = -0.19993

So P'(7) ⁵0 means P(v) is decreasing P'(9)=0.03312

means P(v) is increasing So P'(9) > 0

Therefore, v is a local minimum which is required to minimize power

b) What velocity maximizes the total distance traveled by the pigeon? And how far will the pigeon fly with this velocity?

1- We need to find the derivative of the distance using quotient rule

 $d' = (5*10^4)(17v^-1 + 10^-3*v^-3) - (-17v^-2 + 10^-3) - (-17v^-2) + 10^-3 +$ $(3v^2/1000))(5*10^4)v / (17v^-1 + 10^-3*v^-3)$

2- finding critical points:

When d'=0

V=11.41858 m/s

3-check if the critical point is a maximum using the First Derivative Test:

By picking a number less than v (v=10) and a number greater than v (v=12) ,then substituting them in the formula d'(v)

d'(10)= 9602

So d'(10) > 0 means d(v) is increasing

d'(12)= -1483

So d'(12) < 0 means d(v) is decreasing

Therefore, v is a local maximum which is required to maximize total distance traveled.

C) Show that the velocity that maximizes distance satisfies P'(v) = P(v)/v

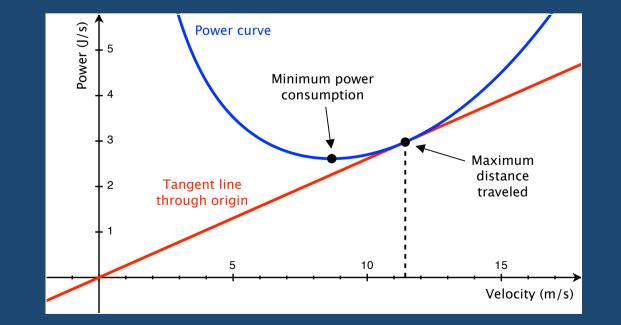
To prove the equation above, we used the same methods of solving as in part (a) except that we used notations instead of numbers. We wrote everything in terms of velocity.

P(v) = E/t(v) so t(v) = E/P(v).

 $D(v) = v \times t(v)$. By substituting t(v) we get:

 $D(v) = v \times [E/P(v)].$

We found the derivative of D (v) by using the quotient rule and we got the equation :



 $P(v) = v \times P'(v)$. Dividing by v we get: P(v)/v = P'(v) which is what we wanted to prove from the first place.

Therefore the maximum distance = 191740 m by substituting v=11.4 m/s in the formula

The slope of P(V)/v is the same as P' as the tangent line passes through the origin and the derivative of the tangent line is the same as the slope of P(V)/v since we can use the slope formula by choosing (0,0) as it passes through the origin and another point we can see that they have the same slope. P (V)/v just means the rise/run since the y axis represents the Power and the x axis represents the velocity and this is the same as the slope of the tangent or the derivative P'.

As we found out from our calculations, there are two different velocities one that minimizes the power and another one that maximizes the distance. After calculating the distance we found it to be a huge number; however, it is still reasonable as we are considering bird migration and birds travel about 4km an hour while the huge number we got was only in meters.

Site used for the tangent graph:

2. The second graph is from Rogawski's calculus book