SOLUTIONS

Quiz #5

1. **(3 points)** The vectors $\vec{u}$ and $\vec{v}$ are three-dimensional vectors that lie in the *xy*-plane. You may assume that the angle between the vectors is 150°, that $|\vec{u}| = 6$ and that $|\vec{v}| = 8$. Find the area of the parallelogram that has $\vec{u}$ and $\vec{v}$ as two of its sides.

$$\text{Area} = |\vec{u} \times \vec{v}|.$$

$$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \sin(\theta)$$

$$= (6)(8) \cdot \sin(150°)$$

$$= 24.$$

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2. In this problem $\vec{u}$, $\vec{v}$ and $\vec{w}$ will always refer to the following vectors:

$$\vec{u} = <1, 2, -3> \qquad \vec{v} = <-7, -14, 21> \qquad \vec{w} = <0, 1, 1>.$$

(a) **(1 point)** Calculate $\vec{v} \times \vec{w}$. Show your work and circle your final answer.

$$\vec{v} \times \vec{w} = \begin{array}{ccccc} i & j & k & i & j \\ -7 & -14 & 21 & -7 & -14 \\ 0 & 1 & 1 & 0 & 1 \end{array} = \langle -35, 7, -7 \rangle$$

(b) **(2 points)** Calculate the volume of the parallelpiped whose sides are formed by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. Show your work and circle your final answer.

$$\text{Volume} = |\vec{u} \bullet (\vec{v} \times \vec{w})|$$

$$= |\langle 1, 2, -3 \rangle \bullet \langle -35, 7, -7 \rangle|$$

$$= |-35 + 14 + 21|$$

$$= 0.$$

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In this problem $\vec{u}$, $\vec{v}$ and $\vec{w}$ will always refer to the following vectors:

$$\vec{u} = \langle 1, 2, -3 \rangle \qquad \vec{v} = \langle -7, -14, 21 \rangle \qquad \vec{w} = \langle 0, 1, 1 \rangle.$$

(c) **(1 point)** Calculate $\vec{u} \times \vec{v}$. Show your work and circle your final answer.

$$\vec{u} \times \vec{v} = \begin{array}{ccccc} i & j & k & i & j \\ 1 & 2 & -3 & 1 & 2 \\ -7 & -14 & 21 & -7 & -14 \end{array}$$

$$= \langle 0, 0, 0 \rangle$$

(d) **(1 point)** What can you conclude about the vectors $\vec{u}$ and $\vec{v}$? Circle any of the statements that you believe to be true. Note that you will lose points for circling statements that are not true.

i. $\vec{u}$ and $\vec{v}$ are parallel (or anti-parallel).

ii. $\vec{u}$ and $\vec{v}$ are both perpendicular to the vector $\vec{v} \times \vec{w}$.

iii. $\vec{u}$ and $\vec{v}$ are orthogonal.

iv. $\vec{u}$ and $\vec{v}$ are unit vectors.

v. $\dfrac{\vec{u}}{|\vec{v}|}$ is a unit vector.

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3. **(2 points)** Find an equation of the form:

$$\langle x,y,z\rangle = \langle x_0,y_0,z_0\rangle + t\cdot\langle a,b,c\rangle$$

for the line that is formed by the intersection of the following planes:

$$x+y-z=2$$

$$3x-4y+5z=6.$$

Show all work and circle your final answer.

Direction vector of line:

$$\langle 1,1,-1\rangle \times \langle 3,-4,5\rangle = \begin{array}{ccccc} i & j & k & i & j \\ 1 & 1 & -1 & 1 & 1 \\ 3 & -4 & 5 & 3 & -4 \end{array}$$

$$= \langle 9,-8,-7\rangle$$

Coordinates of point on intersection:

Assume $z=0$.

$$\begin{aligned} x + y &= 2 \\ 3x - 4y &= 6 \end{aligned}$$

Solution: $x=2$ $y=0$

Coordinates of point are $(2,0,0)$.

Vector equation of line:

$$\langle x,y,z\rangle = \langle 2,0,0\rangle + t\cdot\langle 9,-8,-7\rangle$$