Math 259, Winter 2009. Answers to Unit Test 3 Review Problems – Set B.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) 80.

1.(b) −78.

1.(c) 513/4.

1.(d) −4.5.

1.(e) 1.

1.(f) $(e-1)/2$.

2.(a) Volume = $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}(x+1)\,dydx$.

2.(b) Volume = $\int_{0}^{1}\int_{0}^{2\pi}(1+r\cos(\theta))\cdot r\cdot d\theta dr$.

2.(c) Volume = π.

3.(a) Global max = 7, attained at $(x, y, z) = (36/7, 9/7, 4/7)$. Global min = −7, attained at $(x, y, z) = (-36/7, -9/7, -4/7)$.

3.(b) Global max = 20, attained at $(x, y, z) = (4, 4, 2)$ and at $(x, y, z) = (-4, -4, 2)$. Global min = −20, attained at $(x, y, z) = (-4, 4, -2)$ at $(x, y, z) = (4, -4, -2)$.

3.(c) Global max = 81/4, attained at the eight points $(x, y, z) = (\pm 3 , \pm 3/2, \pm 1)$. Global min = 0, attained on an infinite set of points.

3.(d) There is no global maximum. Global min = 25/3, attained at $(x, y, z) = (-5/3, 1/3, 7/3)$.

3.(e) Global max = $1 + \sqrt{2}$, attained at $(x, y, z) = (-1/\sqrt{2}, -1/\sqrt{2}, 1 + \sqrt{2})$. Global min = $1 - \sqrt{2}$, attained at $(x, y, z) = (1/\sqrt{2}, 1/\sqrt{2}, 1 - \sqrt{2})$.

4.(a) 512/21.

4.(b) 32/3.

4.(c) 4/3.

4.(d) $\int_0^{\pi}\int_0^{y}\frac{\sin(y)}{y}\cdot dxdy = 2.$

4.(e) $\int_0^{1}\int_0^{x}\frac{1}{1+x^4}\cdot dydx = \frac{\pi}{8}.$

5.(a) 18.

5.(b) 128.

5.(c) 1/60.

5.(d) 0.

5.(e) 12.

6.(a) Picture (I).

6.(b) Picture (III).

6.(c) Picture (II).

6.(d) Picture (IV).

7.(a) $\delta = 2\rho - 3.$

7.(b) Mass = $\int_0^{2\pi}\int_0^{\pi}\int_6^{7}(2\rho-3)\cdot\rho^2\cdot\sin(\varphi)\cdot d\rho d\varphi d\theta.$

7.(c) Mass = 1702π g.

8.(a) Radius = $\sqrt{2ah-h^2}$.

8.(b) Volume = $\frac{\pi h\left(3b^2+h^2\right)}{6}.$

9. Volume = $\frac{2}{3}\left(19\pi + 2\sqrt{2} - 54 \cdot \tan^{-1}\left(\sqrt{2}\right)\right)$.

10. Use Lagrange multipliers with $f(x, y, z) = z$ and two constraints: $g(x, y, z) = x + y + z - 12 = 0$ and $h(x, y, z) = x^2 + y^2 - z = 0$. The lowest point is (2, 2, 8) and the highest point is (-3, -3. 18).