Math 259 **Winter 2009**

Unit Test 3 Review Problems – Set B

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. Evaluate each of the double integrals listed below.

(a) $\int_0^2 \int_0^4 (3x + 4y)\,dxdy$ **(b)** $\int_{-1}^2 \int_1^3 (2x - 7y)\,dydx$

(c) $\int_0^3 \int_0^3 (xy + 7x + y)\,dxdy$ **(d)** $\int_{-1}^2 \int_{-1}^2 (2xy^2 - 3x^2y)\,dydx$

(e) $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x)\cos(y)\,dxdy$ **(f)** $\int_0^1 \int_0^1 xe^y\,dydx$

2. Consider the solid bounded by the xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane $z = x + 1$.

(a) Set up a double integral in Cartesian coordinates that gives the volume of the solid.

(b) Convert your answer from Part (a) to a double integral in polar coordinates.

(c) Calculate the volume of the solid.

3. For each of the problems listed below, find the global maximum and global minimum of the function f subject to the constraint $g = 0$.

(a) $f(x, y, z) = x + y + z$ $g(x, y, z) = x^2 + 4y^2 + 9y^2 - 36$

(b) $f(x, y, z) = xy + 2z$ $g(x, y, z) = x^2 + y^2 + z^2 - 36$

(c) $f(x, y, z) = x^2y^2z^2$ $g(x, y, z) = x^2 + 4y^2 + 9z^2 - 27$

(d) $f(x, y, z) = x^2 + y^2 + z^2$ $g(x, y, z) = x + y + z - 1$

and $h(x, y, z) = x + 2y + 3z - 6 = 0$

(e) $f(x, y, z) = z$ $g(x, y, z) = x + y + z - 1$

and $h(x, y, z) = x^2 + y^2 - 1 = 0$

4. Evaluate each of the double integrals listed below. If a double integral seems unusually difficult to find, try changing the order of integration.

(a) $\int_{-2}^{2}\int_{x^2}^{4} x^2 y \cdot dydx$ **(b)** $\int_{-1}^{3}\int_{x^2}^{2x+3} x \cdot dydx$

(c) $\int_{0}^{2}\int_{2x}^{4x-x^2} 1 \cdot dydx$ **(d)** $\int_{0}^{\pi}\int_{x}^{\pi} \frac{\sin(y)}{y} \cdot dydx$

(e) $\int_{0}^{1}\int_{y}^{1} \frac{1}{1+x^4} \cdot dxdy$

5. In each part of this problem you are given a function $f(x, y, z)$ and a three-dimensional region S. In each case, compute the value of the triple integral:

$$\iiint_S f(x,y,z)dV.$$

(a) $f(x, y, z) = x + y + z$.
The region S is the rectangular prism $[0, 2]\times[0, 3]\times[0, 1]$.

(b) $f(x, y, z) = xyz$.
The region S is the rectangular prism $[-1, 3]\times[0, 2]\times[-2, 6]$.

(c) $f(x, y, z) = x^2$.
The region S is the part of the first octant bounded by the coordinate planes and $x + y + z = 1$.

(d) $f(x, y, z) = xyz$.
The region S is bounded by $z = 1 - x^2$, $x = -1$, $x = 1$, $y = 0$, $y = 2$ and the xy plane.

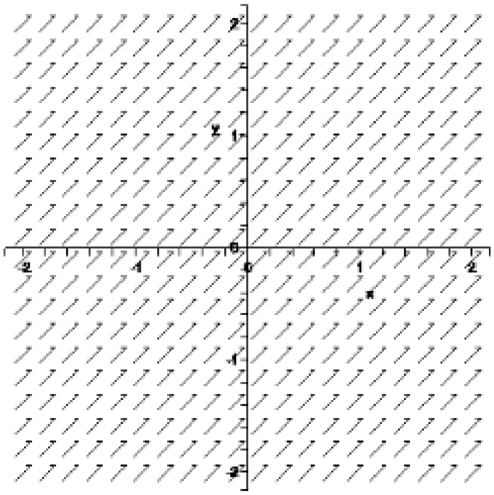
(e) $f(x, y, z) = x + y$.
The region S is bounded by $z = 2 - x^2$, $z = x^2$, $y = 0$ and $y = 3$.

6. The next pages shows pictures of four vector fields. Match each of the formulas given below with one of the pictures given on the next page.

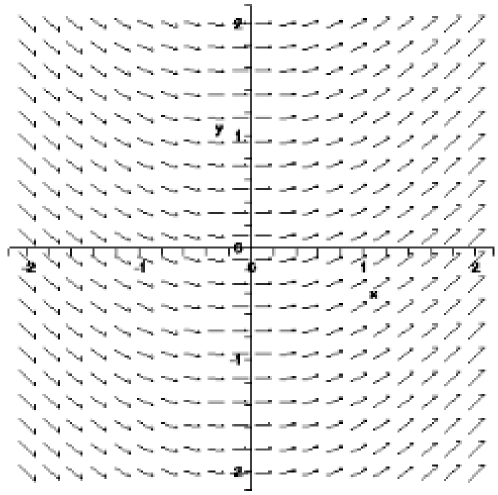
(a) $\vec{F}(x,y) = <1,1>$ **(b)** $\vec{F}(x,y) = <x,-y>$

(c) $\vec{F}(x,y) = \langle 2, x \rangle$ **(d)** $\vec{F}(x,y) = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$.

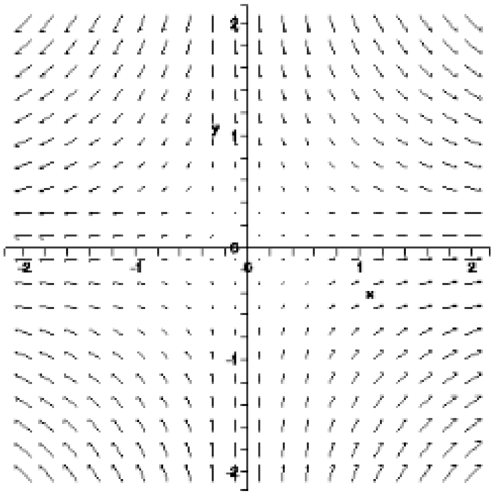
(I)



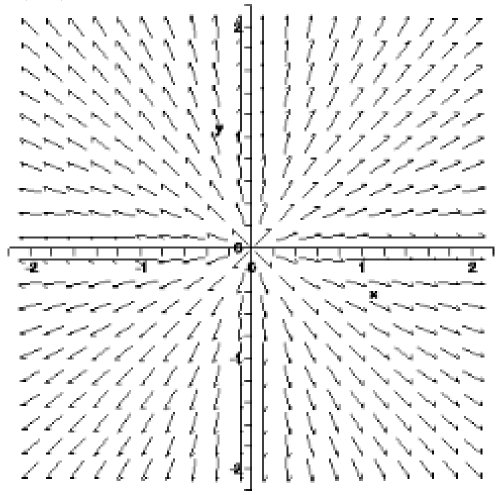
(II)



(III)



(IV)



7. A spherical shell centered at the origin has an inner radius of 6cm and an outer radius of 7cm. The density of the shell increases linearly with the distance from the center of the sphere. At the inner surface, $\delta = 9$ g/cm^3 and at the outer surface $\delta = 11$ g/cm^3.

(a) Find a formula for the density δ as a function of $\rho = (x^2 + y^2 + z^2)^{1/2}$.

(b) Write down a triple integral that gives the mass of the spherical shell.

(c) Calculate the mass of the spherical shell.

8. Let a and h be positive constants with $0 < h < a$. Consider the three dimensional region K bounded by the surfaces:

- from above by $x^2 + y^2 + z^2 = a^2$
- from below by $z = a - h$.

(a) The "shadow" that K casts in the xy-plane is a circle. Find the radius of this circle. (Your answer may contain a, h and other constants.)

(b) Find the volume of the region K. (Your answer may contain a, h and other constants.)

9. Consider the three-dimensional region S bounded by the following surfaces:

- $x = -1$
- $x = 1$
- $y = -1$
- $y = 1$
- $x^2 + y^2 + z^2 = 4$.

Find the volume of the region S. **HINT:**

$$\int_0^1 \left(4\sqrt{3 - x^2} + 4\left(4 - x^2\right) \cdot \sin^{-1}\left(\frac{1}{\sqrt{4 - x^2}} \right) \right) dx = \frac{2}{3}\left(19\pi + 2\sqrt{2} - 54 \cdot \tan^{-1}\left(\sqrt{2}\right)\right).$$

10. The plane $x + y + z = 12$ intersects the paraboloid $z = x^2 + y^2$. The curve of intersection is an ellipse, as shown in the diagram given below. Find the lowest (i.e. lowest value of z) point and highest point on this ellipse.