Math 259, Winter 2009. Answers to Unit Test 2 Review Problems – Set B.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $4x + 2y + z = 8$.

1.(b) $10x - 16y - z = 9$.

1.(c) $-3x - y + z = 6$.

1.(d) $z = -1$.

2.(a) The partial derivative $f_x(x, y)$ is positive (at least over the region shown).

2.(b) The partial derivative $f_y(x, y)$ is negative (at least over the region shown).

2.(c) (I) $f(2, 1) = 10$.
(II) $f_x(2, 1) \approx 2$.
(III) $f_y(2, 1) = -4$.

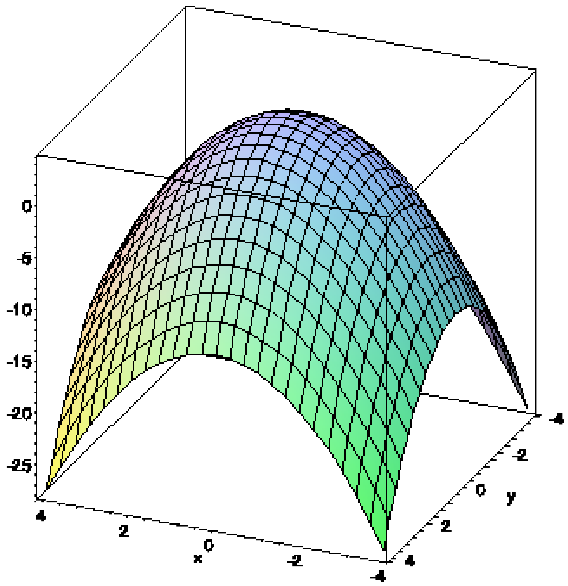
3.(a) $$f_T(500,24) \approx \frac{f(520,24) - f(500,24)}{520-500} = 0.0255.$$

3.(b) $$f_P(500,24) \approx \frac{f(500,26) - f(500,24)}{26-24} = -0.915.$$

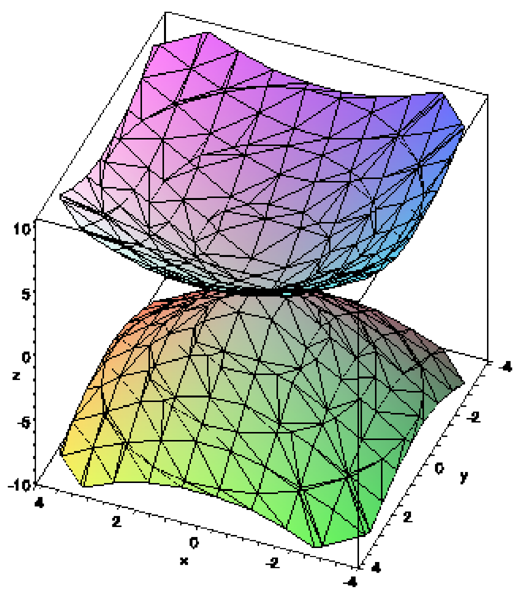
3.(c) $V \approx 23.69 + 0.0255(T - 500) - 0.915(P - 24)$.

3.(d) $V \approx 23.69 + 0.0255(505 - 500) - 0.915(24.3 - 24) = 23.54 \text{ ft}^3$.

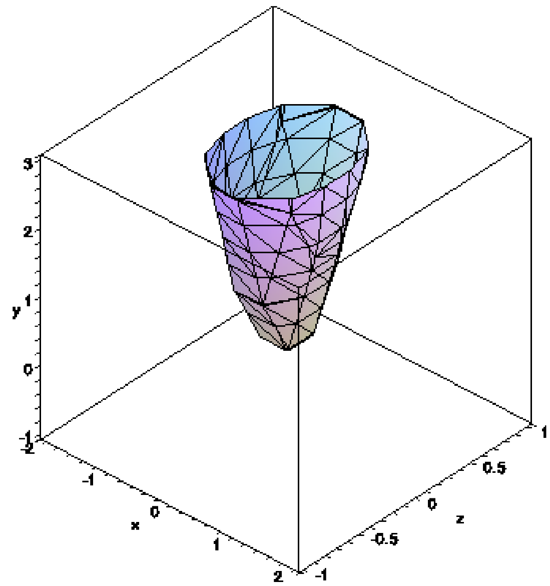
4.(a) The surface is an elliptic paraboloid that intersects the z-axis at the point (0, 0, 4) and then opens downwards. The cross-sections of the surface parallel to the xy-plane are circles centered on the origin.



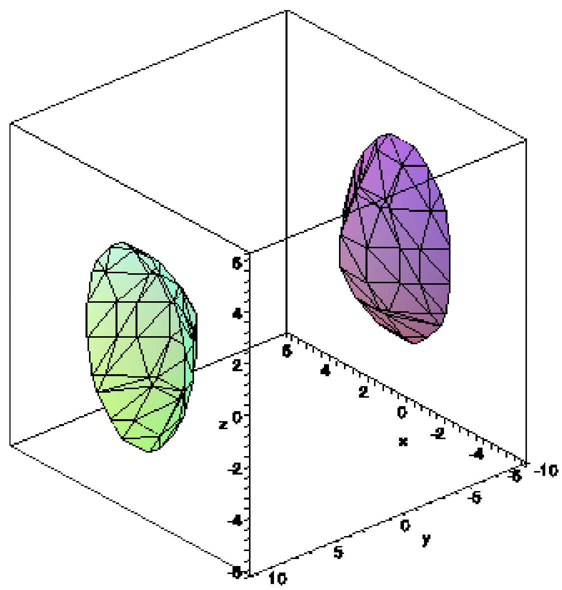
4.(b) The surface is a cone that opens along both the positive and negative *z*-axes. The point of the cone is located at the origin.



4.(c) The surface is an elliptic paraboloid that opens along the positive *y*-axis. The cross-sections parallel to the *xz*-plane are ellipses.



4.(d) The surface is a hyperboloid of two sheets that opens along the positive and negative *y*-axes. The cross-sections parallel to the *xz*-plane are ellipses. The points where the hyperboloid intersects the *y*-axis are the points (0, 6, 0) and (0, −6, 0).



5.(a) There is a saddle point located at (0, 0, 0), a local minimum located at (−1, −1, −2) and another local minimum located at (1, 1, −2).

5.(b) There is a saddle point located at (−1, 1, 5) and a local minimum located at (3, −3, −27).

5.(c) There is a saddle point located at (0, −2, 32) and a local minimum located at (−5, 3, −93).

5.(d) There is a saddle point located at (0, 0, 0), a local maximum located at (1, 2, 2) and another local maximum located at (−1, −2, 2).

6. The cultists should raise 40 hogs and 40 cattle. The cultists should raise no sheep.

7. Each triangular end should have a height of $h = (V/\sqrt{2})^{1/3}$ and a base of $b = 2h$. The distance between the front and the back of the house should be $(\sqrt{2})h$.

8. The maximum error is 0.022 acres.

9.(a) The limit does not exist. A good strategy to use here is to substitute $y = mx$ into the expression and take the limit as $x \to 0$. The expression you get depends on m.

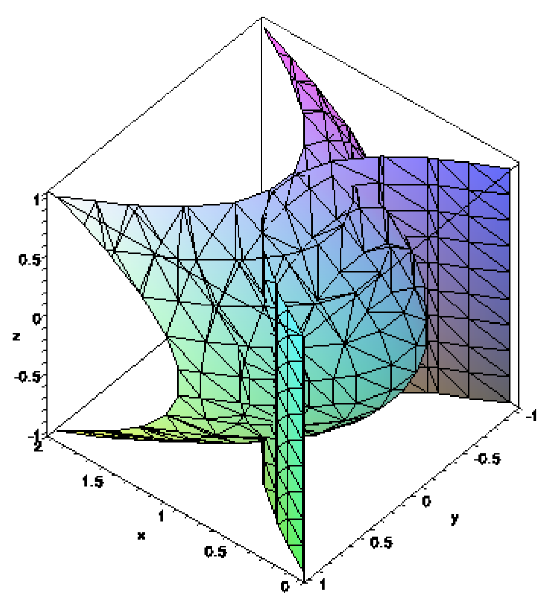
9.(b) The limit does not exist. A good strategy to use here is to substitute $y = mx$ into the expression and take the limit as $x \to 0$. The expression you get depends on m.

9.(c) The limit does not exist. A good strategy to use here is to substitute $y = mx$ into the expression and take the limit as $x \to 0$. The expression you get depends on whether m is positive or negative.

9.(d) The limit does not exist, however it is not easy to show this by only considering straight lines that approach (0, 0). For this, try the curves $y = x$ and $y = x^2$ to approach (0, 0).

10.(a) The surface $x = 1 - y^2$ is a parabolic cylinder that extends along the z-axis and the surface $x = y^2 + z^2$ is an elliptic paraboloid that opens along the positive x-axis. The cross-sections of this elliptic paraboloid parallel to the yz-plane are circles.

10.(b) The graphs of the two surfaces are shown below.



10.(c) There are many possible formulas for this vector functions. Here is one of them.

$$\vec{r}(t) = <1 - \tfrac{1}{2}\cos^2(t), \tfrac{1}{\sqrt{2}}\cos(t), \sin(t)>$$

10.(d) The parametric equations for the projection of the curve onto the *yz*-plane are:

$y(t) = (1/\sqrt{2})\cos(t)$ and $z(t) = \sin(t)$.

We can use these to create a Cartesian equation for this projection using the trigonometric identity $\sin^2(t) + \cos^2(t) = 1$:

$$\frac{y(t)^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{z(t)^2}{1^2} = \cos^2(t) + \sin^2(t) = 1.$$

This is an equation for an ellipse.

11. Step 1:

$$\left|f(x,y) - L\right| = \left|\frac{x^2y}{x^2+y^2}\right|.$$

Step 2:

Note that:

(I) $x^2 \le x^2 + y^2$ so that $\dfrac{x^2}{x^2+y^2} \le 1$.

(II) $y^2 \le x^2 + y^2$ so that $|y| \le \sqrt{x^2+y^2}$.

Using (I) and (II) gives that:

$$\left|\frac{x^2y}{x^2+y^2}\right| = \frac{x^2}{x^2+y^2} \cdot |y| \le |y| \le \sqrt{x^2+y^2}.$$

Step 3:

Let $\varepsilon > 0$ be given. Then let $\delta = \varepsilon$.