**Math 259** **Winter 2009**

## Unit Test 2 Review Problems – Set B

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

**1.** Find an equation for the tangent plane at the point given for each of the surfaces $z = f(x, y)$ listed below.

**(a)** **Surface:** $z = 5 - 2x^2 - y^2$

**Point:** $(1, 1, 2)$

**(b)** **Surface:** $z = x^2 - 4y^2$

**Point:** $(5, 2, 9)$

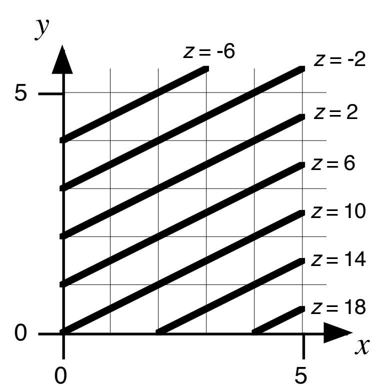
**(c)** **Surface:** $z = e^y + x + x^2 + 6$

**Point:** $(1, 0, 9)$

**(d)** **Surface:** $z = \sin(xy)$

**Point:** $(2, 3\pi/4, -1)$

**2.** The diagram given below shows the contour diagram for a function $z = f(x, y)$. Use this diagram to answer the questions listed below.




**(a)** Is the partial derivative $f_x(x, y)$ positive or negative?

**(b)** Is the partial derivative $f_x(x, y)$ positive or negative?

**(c)** Estimate each of the following quantities:

(I) $f(2, 1)$ (II) $f_x(2, 1)$ (III) $f_y(2, 1)$

**3.** The volume of one pound (i.e. basically a fixed mass of steam in the sense that in a constant gravitational field, pounds and kilograms are closely related) of steam depends on the pressure and the temperature. If temperature is measured in °F, volume in cubic feet and pressure in pounds per square foot then the volume of one pound of steam is given in the table shown below.

| | $P = 20$ lb/ft$^2$ | $P = 22$ lb/ft$^2$ | $P = 24$ lb/ft$^2$ | $P = 26$ lb/ft$^2$ |
|---|---|---|---|---|
| $T = 480$°F | 27.85 | 25.31 | 23.19 | 21.39 |
| $T = 500$°F | 28.46 | 25.86 | 23.69 | 21.86 |
| $T = 520$°F | 29.06 | 26.41 | 24.20 | 22.33 |
| $T = 540$°F | 29.66 | 26.95 | 24.70 | 22.79 |

In this problem we will use the notation $V = f(T, P)$ to denote volume ($V$) as a function of temperature ($T$) and pressure ($P$).

**(a)** Approximate the value of the partial derivative $f_T(500, 24)$.

**(b)** Approximate the value of the partial derivative $f_P(500, 24)$.

**(c)** Write down a formula for the linear approximation of $f(T, P)$ to the point $(T, P) = (500, 24)$.

**(d)** Approximate the value of $f(505, 24.3)$.

**4.** For each of the equations listed below, interpret the equation as a surface in 3D. Describe the surface and draw a sketch of its graph.

**(a)** $z = 4 - x^2 - y^2$ **(b)** $z^2 = 4(x^2 + y^2)$

**(c)** $y = 4x^2 + 9z^2$ **(d)** $y^2 - 9x^2 - 4z^2 = 36$

**5.** Find and classify all critical points for each of the surfaces listed below.

**(a)** $f(x, y) = x^4 + y^4 - 4xy$.

**(b)** $f(x, y) = x^3 + 6xy = 3y^2 - 9x$.

**(c)** $f(x, y) = 3x^2 + 6xy + 2y^3 + 12x - 24y$.

**(d)** $f(x, y) = 4xy - 2x^4 - y^2$.

**6.** An swine-worshipping agricultural cult plans to raise hogs, sheep and cattle in their compound. The cult is interested in making as much profit from their activities as possible they want to raise the combination of hogs, sheep and cattle that will maximize profits. Based on prevailing prices, the cultists anticipate a profit of \$100 per sheep, \$80 per hog and \$200 for each cow. The cult compound will accommodate 80 sheep, 120 hogs or 60 cattle (or any reasonable combination; 8 sheep use the same amount of cult resources as 12 hogs or 6 cattle). The cult's belief system requires them to raise as many hogs as sheep and cattle combined. How many sheep, hogs and cattle should the cult raise?

**7.** An A-frame house (see picture below) is built with a fixed volume $V > 0$. The front and the back of the house are parallel, isosceles triangles with equal area. The roof of the house consists of two rectangles that connect to the long sides of the triangles. To minimize heating costs, the total surface of the house (excluding the floor) is to be minimized. Describe the shape of the house that will achieve this.

**8.** A surveyor wants to find the area of a triangular field that she has been contracted to measure. Her contract specifies that the area of the field should be given in acres (1 acre is equal to 43560 square feet). The surveyor measures two sides of the field and finds they are 500 feet and 700 feet long. She estimates that there may be an error of 1 foot in each of these measurements. The surveyor measures the angle between the two sides to be $30^{\circ}$, and her equipment is rated to measure angles accurately to within $0.25^{\circ}$. Use differentials to estimate the maximum possible error (expressed in acres) of the surveyor's estimate of the area of the triangular field.

**HINT:** The area of a triangle with sides $a$ and $b$ with angle $\theta$ between them is:

$$Area = 0.5 \cdot a \cdot b \cdot \sin(\theta).$$

**9.** For each of the limits listed below, either find the value of the limit or show that the limit does not exist.

**(a)** $$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$$

**(b)** $$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

**(c)** $$\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{|x\cdot y|}$$

**(d)** $$\lim_{(x,y)\to(0,0)} \frac{x^2\cdot y}{x^4+y^2}$$

**10.** Consider the two surfaces defined by the equations:

$$x = 1 - y^2 \qquad \text{and} \qquad x = y^2 + z^2.$$

**(a)** Classify each of the two surfaces.

**(b)** Using one set of axes, sketch the two surfaces.

**(c)** Find a vector equation for the curve formed by the intersection of the two surfaces.

**(d)** Show that the projection of the curve of intersection onto the *yz*-plane is an ellipse.

**11.** Use epsilon (ε) and delta (δ) to prove that the following limit has the values given below.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + y^2} = 0.$$