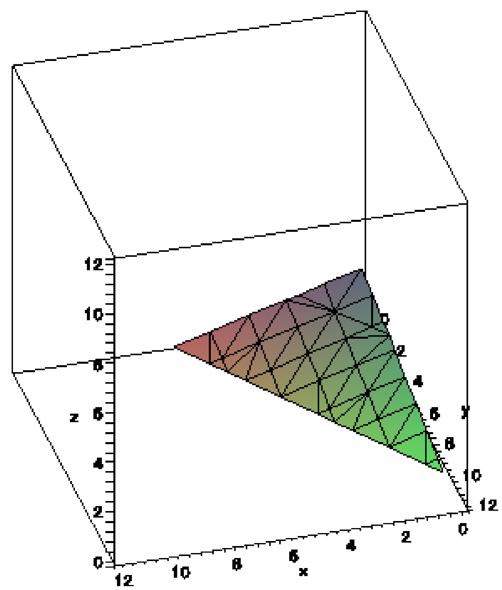
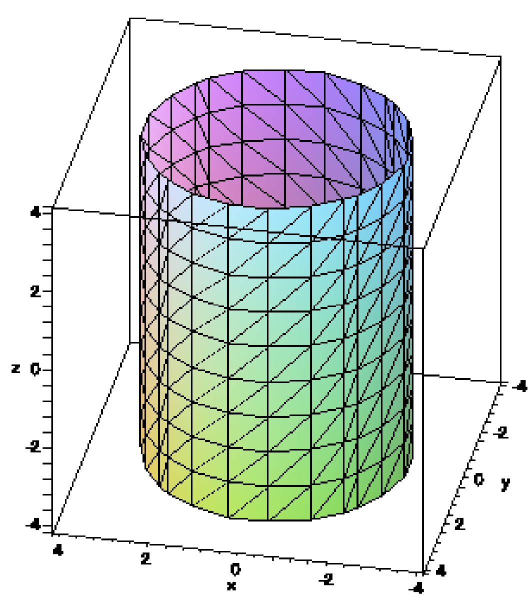
Math 259, Winter 2009. Answers to Unit Test 2 Review Problems – Set A.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

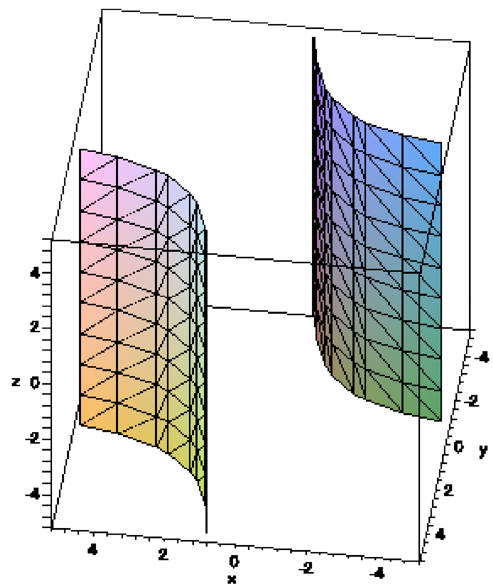
1.(a) This is the equation of a plane with intercepts located at (20/3, 0, 0), (0, 10, 0) and 0, 0, 2). A graph of this plane is shown below.



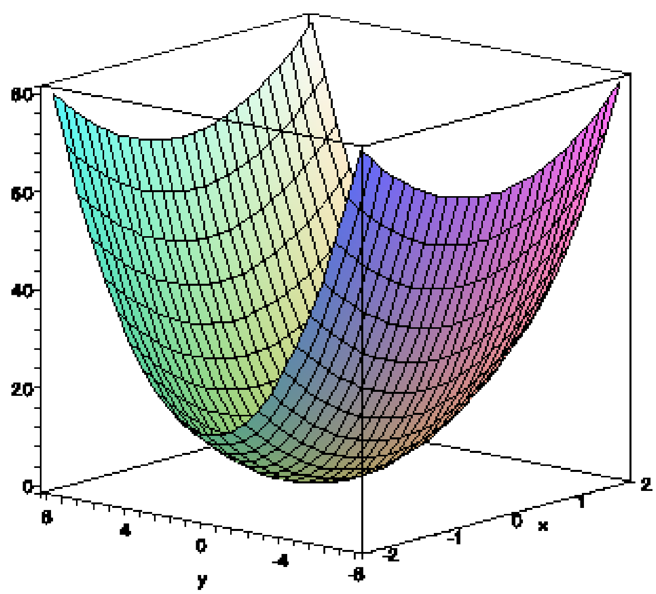
1.(b) This is the equation of a cylinder that extends along the *z*-axis. The cross-sections of the cylinder parallel to the *xy*-plane are circles of radius 3 centered on the origin.



1.(c) This is the equation of a cylinder that extends along the z-axis. The cross-sections parallel to the xy-plane is a hyperbola.

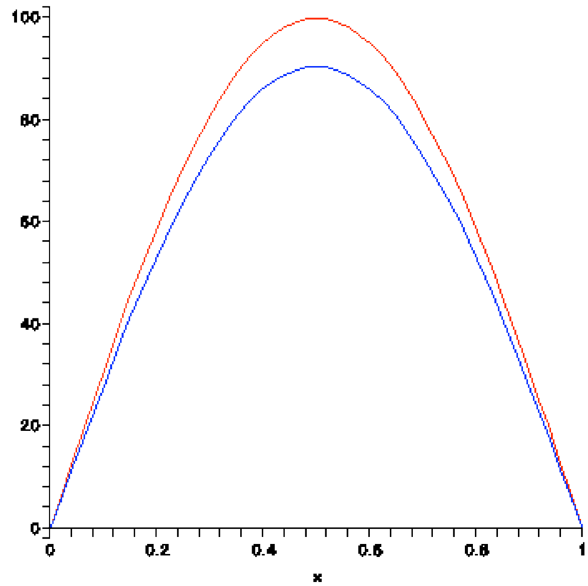


1.(d) This is the equation of an elliptic paraboloid that opens along the positive z-axis. The cross-sections parallel to the xy-plane are ellipses.



2. None of the limits exist. Implement the suggested strategies to see that this is the case for each part of the problem.

3.(a) The graphs of $H(x, t)$ for $t = 0$ and $t = 1$ are shown below. The red curve corresponds to $t = 0$ and the blue curve corresponds to $t = 1$.



3.(b) $H_x(0.2, t) = 254.2e^{-0.1t}$ °C/m. The practical interpretation of this partial derivative is that if the position on the metal bar was increased from $x = 0.2$ to $x = 1.2$ at the instant of time t then the temperature would increase by approximately $254.2e^{-0.1t}$ °C. The sign makes sense because (as the above graphs show) when $x = 0.2$, the slope of the tangent line is positive.

3.(c) $H_x(0.8, t) = -254.2e^{-0.1t}$ °C/m. The practical interpretation of this partial derivative is that if the position on the metal bar was increased from $x = 0.8$ to $x = 1.8$ at the instant of time t then the temperature would decrease by approximately $254.2e^{-0.1t}$ °C. The sign makes sense because (as the above graphs show) when $x = 0.8$, the slope of the tangent line is negative.

3.(d) $H_t(x, t) = -10e^{-0.1t}\sin(\pi x)$ °C/minute. The sign is negative. This makes sense because (assuming that the surroundings are cooler than the bar) when the heat source is removed, the bar as a whole will start to cool down.

4.(a) $6x + 8y - z = 25$.

4.(b) $z = -1$.

4.(c) $27x - 12y - z = 38$.

4.(d) $x - y + z = 1$.

5.(a) $<0.25, 0.375>$

5.(b) $z = 4 + 0.25(x - 1) + 0.375(y - 4)$

5.(c) Approximately 3.995.

6.(a) 34/3°C/m.

6.(b) $\mathbf{u}$ = <4/13, 3/13. 12/13>.

6.(c) 13°C/m.

7.(a) There is a local minimum located at (–1, 2, –1).

7.(b) There is a saddle point located at (–0.5, –0.5, 29/4).

7.(c) There is a local minimum located at (–3, 4, –9).

7.(d) There is a saddle point located at (0, 0, 3) and a local maximum at (–1, –1, 4).

8.(a) Use implicit differentiation of the equation of state to obtain the partial derivatives $\partial V/\partial p$ and $\partial V/\partial T$, then calculate $\partial p/\partial V$ and $\partial p/\partial T$.

8.(b) A 5°C increase in temperature will result in a pressure increase of approximately 229.77 atmospheres, which will cause the thermometer to break.

9.(a) The rate of change of the volume is –2880 cubic inches per hour.

9.(b) The rate of change of the volume is 26π/5 cubic feet per minute.

10.(a) There are many ways to write down the vector function for this curve. The approach is to rearrange both equations for one variable (I used z), then equate the two equations, and try to recognize the equation that you obtain as one that you know how to write parametric equations for. In this case, the Cartesian equation connecting x and y is the equation of a circle with center (0, 0.5) and radius 0.5.

$$\vec{r}(t) = <\tfrac{1}{2}\cos(t), \tfrac{1}{2}\sin(t) + \tfrac{1}{2}, \tfrac{1}{2}\sin(t) + \tfrac{1}{2}>.$$

10.(b) The projection of this curve onto the xy-plane consists of the functions in the first two components of the vector function. The parametric equations for this projection onto the xy-plane are:

$x(t) = 0.5\cos(t)$ and $y(t) = 0.5\sin(t) + 0.5$.

The Cartesian equation connecting these parametric equations is:

$$x^2 + (y - 0.5)^2 = 0.25,$$

which is the equation of a circle.