Math 259, Winter 2009. Answers to Unit Test 1 Review Problems – Set B.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) Graph (I).

1.(b) Graph (II).

1.(c) Graph (III).

1.(d) Graph (IV).

2.(a) $(\pi - 2)/2$.

2.(b) $\dfrac{39\sqrt{3} - 10\pi}{6}$.

2.(c) 2.

2.(d) $5\pi/4$.

3.(a) Let T_1 be the magnitude of tension in the left cable and T_2 be the magnitude of tension in the right cable. Then $T_1 = T_2 = 50\cdot\sqrt{2} \approx 70.71$ pounds of tension in each cable.

3.(b) Let T_1 be the magnitude of tension in the left cable and T_2 be the magnitude of tension in the right cable. Then $T_1 \approx 96.12$ pounds and $T_2 \approx 71.97$ pounds of tension.

3.(c) Let T_1 be the magnitude of tension in the left cable and T_2 be the magnitude of tension in the right cable. Then $T_1 \approx 85.64$ pounds and $T_2 \approx 64.91$ pounds of tension.

4.(a) π.

4.(b) $3\pi/2$.

4.(c) $9\pi/2$.

4.(d) 4π.

5.(a) The two curves intersect at the point where $x = 1$ and $y = 0$.

5.(b) The two curves intersect at the origin, at the point with $\theta = \pi/6$ and $r = ½$, at the point with $\theta = 5\pi/6$ and $r = ½$, and at the point with $\theta = \pi$ and $r = 1$.

5.(c) The two curves intersect at the origin, at the point with $\theta = \pi$ and $r = 2$, and at the two points where $r = 2(\sqrt{2} - 1)$ and $\theta = \pm\cos^{-1}(3 - 2\sqrt{2})$

5.(d) The two curves intersect at the origin and at the point with $\theta = \pi/4$ and $r = 1/\sqrt{2}$

6.(a) $y = 7$.

6.(b) $7x + 11y = 114$.

6.(c) $3x + 4y - z = 0$.

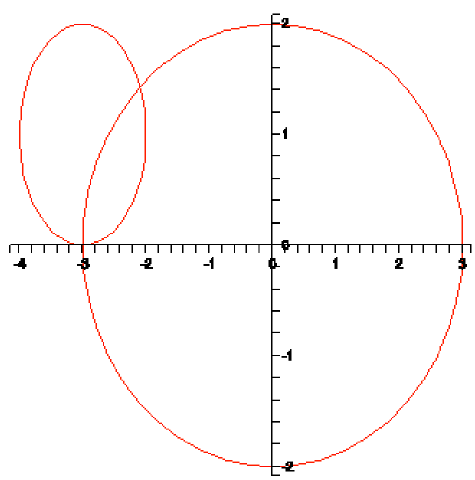
6.(d) $2x - y - z = 0$.

7.(a) The area is approximately 25.229.

7.(b) The volume is 55.

7.(c) No, the plane containing P, Q, R does not contain the origin. If it did, then the volume of the parallelepiped from Part (b) would have been zero.

8.(a) A graph showing the two paths is shown below.



8.(b) There is one collision point, which occurs for $t = 3\pi/2$. The x and y coordinates of the point where the collision takes place are $(-3, 0)$.

8.(c) There are still two intersection points between the paths, but this time there are no collision points.

9. Area $= 2\pi r^2 + \pi d^2$.

10. The area is only really feasible to calculate using polar coordinates. You can rewrite the equation of the folium in polar coordinates to get something along the lines of:

$$r = \frac{3 \cdot \sec(\theta) \cdot \tan(\theta)}{1 + \tan^3(\theta)}.$$

The limits of integration are 0 to $\pi/2$, so the area of the loop is given by:

$$\text{Area} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\frac{3 \cdot \sec(\theta) \cdot \tan(\theta)}{1 + \tan^3(\theta)} \right)^2 d\theta = \frac{3}{2}.$$

To evaluate the integral, try the u-substitution $u = \tan(\theta)$.