Unit Test 1 Review Problems – Set B

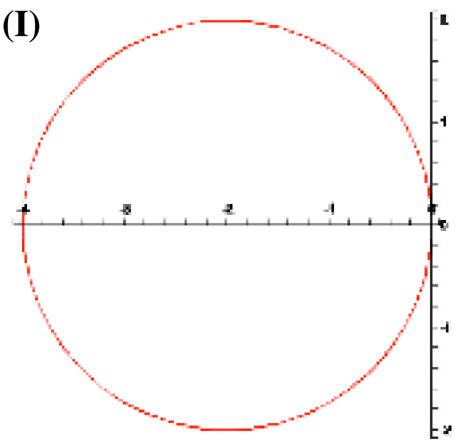
We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. For each of the polar curves given below, match the polar curve with the corresponding graph WITHOUT using a calculator or computer to graph the polar curve. (Hint: Try converting each polar curve to Cartesian coordinates.)

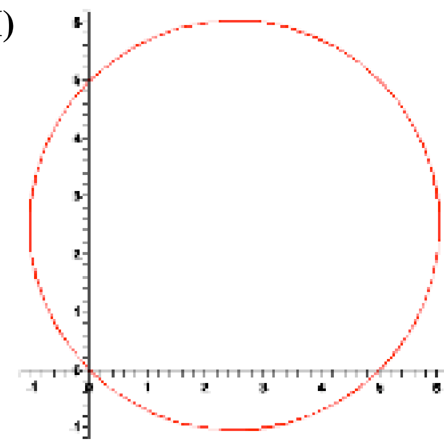
(a) $r = -4\cdot\cos(\theta)$ **(b)** $r = 5\cdot\cos(\theta) + 5\cdot\sin(\theta)$

(c) $r = -4\cdot\cos(\theta) + 3\cdot\sin(\theta)$ **(d)** $r = 8\cdot\cos(\theta) - 15\cdot\sin(\theta)$

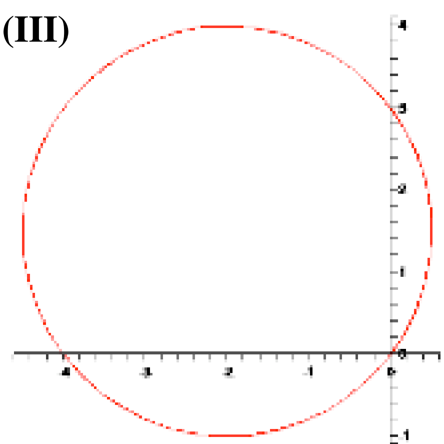
(I)



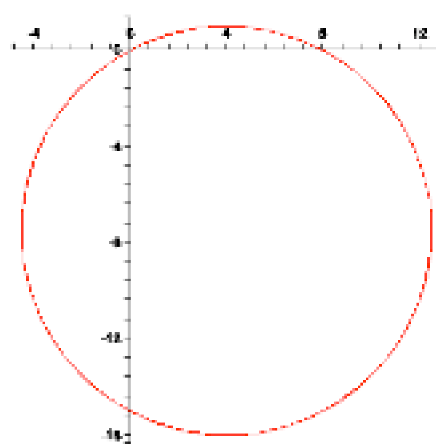
(II)



(III)



(IV)



2. In each case find the area of the region described. It may be helpful to graph the curves, and be certain to pay careful attention to words like "inside" and "outside" in the description of each region.

(a) Inside both $r = 2\cdot\sin(\theta)$ and $r = 2\cdot\cos(\theta)$.

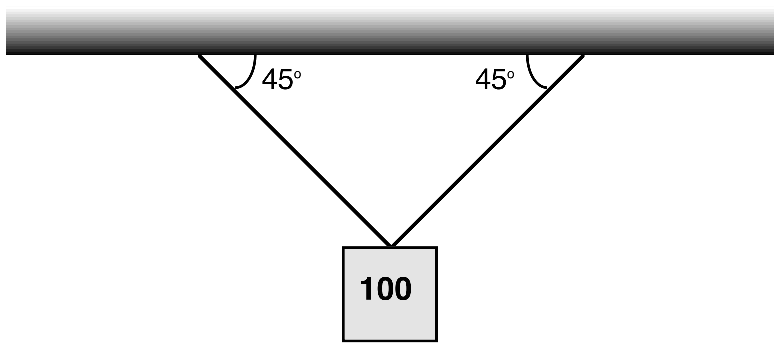
(b) Inside $r = 3 - 2\cdot\sin(\theta)$ and outside $r = 4$.

(c) Inside $r = 2\cdot\sin(2\theta)$ and outside $r = \sqrt{2}$.

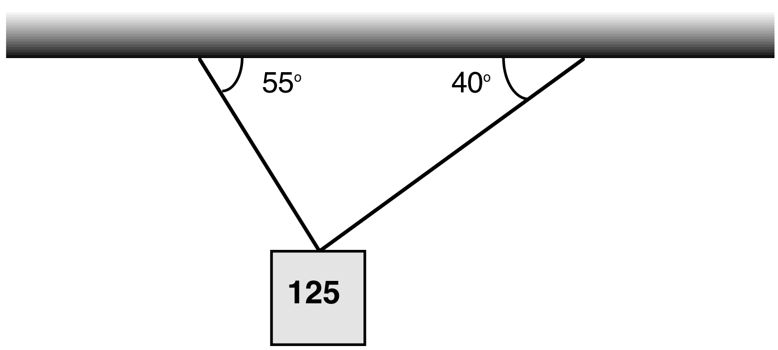
(d) Inside $r = 1 + \cos(\theta)$ and outside $r = \cos(\theta)$.

3. Each of the following diagrams show a weight suspended by two cables. The weights are given in pounds (which is a unit of force, not of mass, so you don't have to multiply by 9.8 m/s^2). In each case, find the magnitude of the tension in each of the cables.

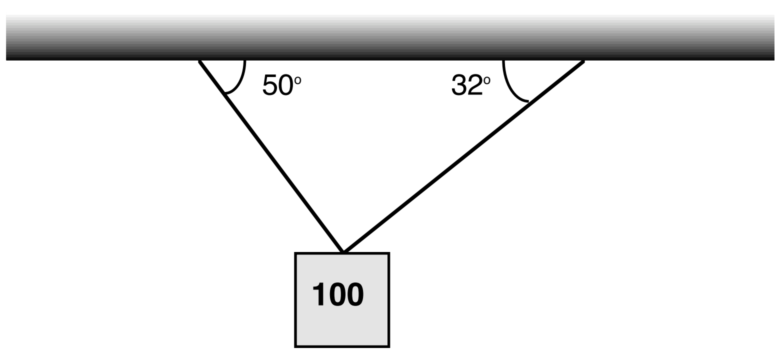
(a)



(b)



(c)



4. In each case, find the area bounded by the polar curve. (It is a really good idea to graph the curves first on your graphing calculator.)

(a) $r = 2\cdot\cos(\theta)$

(b) $r = 1 + \cos(\theta)$

(c) $r = 2 - \cos(\theta)$

(d) $r = -4\cdot\cos(\theta)$.

5. Find all points of intersection between the pairs of curves listed below. (Note that graphing the curves on your calculator might be a good place to start as this will help you to see where the intersection points occur. However, you should calculate the intersection points using functions and algebra, not by tracing on your calculator screen.)

(a) $r = 1$ and $r = \cos(\theta)$.

(b) $r = \sin(\theta)$ and $r = \cos(2\theta)$

(c) $r = 1 - \cos(\theta)$ and $r^2 = 4\cdot\cos(\theta)$

(d) $r = \sin(\theta)$ and $r = \cos(\theta)$.

6. In each case, write an equation for the plane (located in three-dimensional space) that is described.

(a) The plane includes the point $(5, 7, -6)$ and is parallel to the xz-plane.

(b) The plane includes the point $(10, 4, -3)$ with normal vector $<7, 11, 0>$.

(c) The plane includes the origin and is parallel to the plane with equation:

$$3x + 4y = z + 10.$$

(d) The plane includes the three points: $(1, 1, 1)$, $(1, -1, 3)$ and $(0, 0, 0)$.

7. In this problem, P, Q and R will always refer to the following points in three-dimensional space:

$P = (1, 3, -2)$ $\qquad$ $Q = (2, 4, 5)$ $\qquad$ $R = (-3, -2, 2)$.

(a) Find the area of the triangle with vertices at P, Q and R.

(b) Imagine the lines that connect the points P, Q and R to the origin, $O = (0, 0, 0)$. Find the volume of the parallelpiped that has these lines as its edges.

(c) Is it possible that the plane containing the points P, Q, and R also contains the origin? Briefly explain how you can answer this question without making any additional calculations.

8. Suppose that the position of one particle at time t is given by the parametric equations:

$$x_1 = 3 \cdot \sin(t) \qquad y_1 = 2 \cdot \cos(t) \qquad 0 \le t \le 2\pi,$$

and the position of a second particle is given by:

$$x_2 = -3 + \cos(t) \qquad y_1 = 1 + \sin(t) \qquad 0 \le t \le 2\pi,$$

(a) Graph the paths of both particles. How many times do the paths cross?

(b) Do the particles ever collide? If so, determine the smallest value of t for which this occurs.

(c) What happens if the path of the second particle is given by the following parametric equations instead of those listed above?

$$x_2 = 3 + \cos(t) \qquad y_1 = 1 + \sin(t) \qquad 0 \le t \le 2\pi,$$

9. A trochoid is a curve that can be described using the parametric equations:

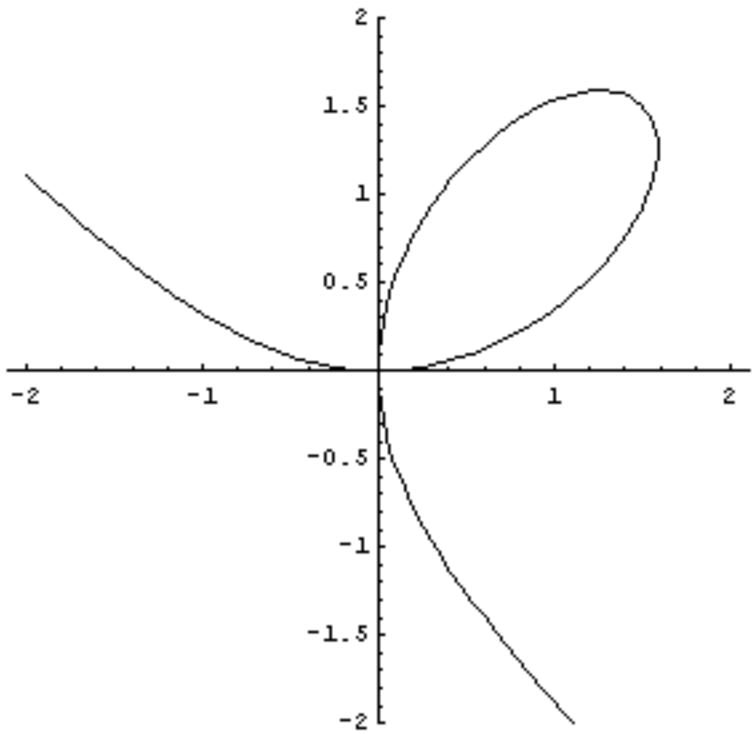
$$x = r \cdot \theta - d \cdot \sin(\theta) \qquad y = r - d \cdot \cos(\theta)$$

where r and d are constants. When $d < r$, the graph of the trochoid in the xy-plane looks like a series of arches (or a wave turned upside down). Assuming $d < r$, find the area beneath one arch of the trochoid.

10. The folium of Descartes is the curve in the xy-plane with the Cartesian equation:

$$x^3 + y^3 = 3xy.$$

A plot of this folium is shown below.



Find the area enclosed by the loop of the folium that occupies the first quadrant of the *xy*-plane. (Hint: Are Cartesian coordinates the most convenient to use here?)