Math 259, Winter 2009. Answers to Unit Test 1 Review Problems – Set A.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $x = 2$ and $r = 2/\cos(\theta)$.

1.(b) $x + y = 1$ and $r = \dfrac{1}{\cos(\theta) + \sin(\theta)}$.

1.(c) $x^2 + y^2 + 8y = 0$ and $r = -8\cdot\sin(\theta)$.

1.(d) $x^2 - 6x + y^2 - 8y = 0$ and $r = 6\cdot\cos(\theta) + 8\cdot\sin(\theta)$.

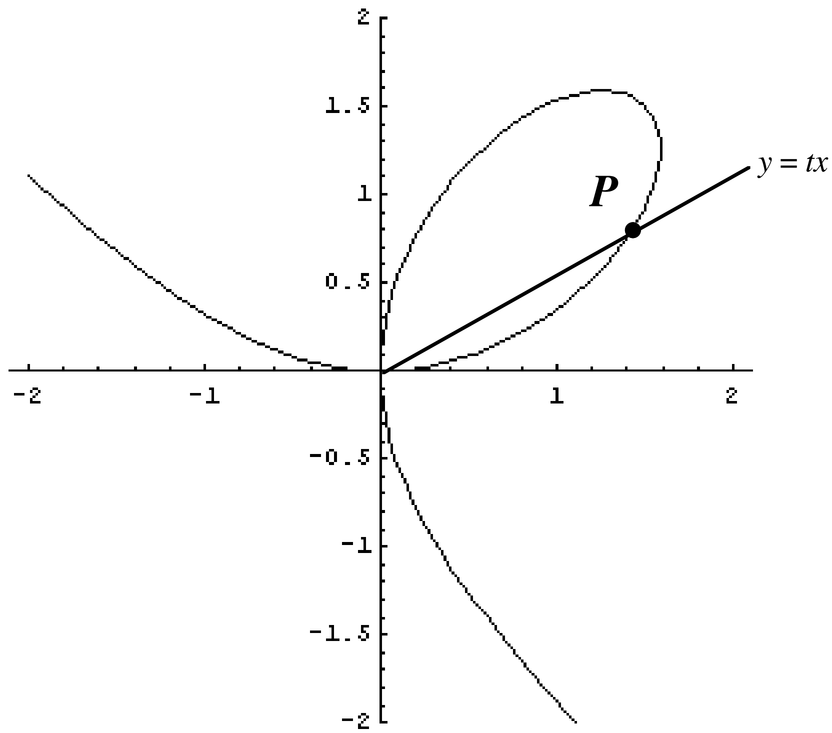
2.(a) (i) The tangent line is horizontal at (1, 2) and (1, –2).
(ii) The tangent line is vertical at (0, 0).
(iii) The x-intercept is at (3, 0). The graph crosses this point twice, once with slope $m = \sqrt{3}$, and then again with slope $m = -\sqrt{3}$.

2.(b) (i) The tangent line is horizontal at the points where $r = 1.5$ and $\theta = \pm\pi/3$.
(ii) The tangent line is vertical where $r = 2$ and $\theta = 0$. The tangent line is also vertical where $r = 0.5$ and $\theta = 5\pi/6$ or $7\pi/6$.
(iii) One x-intercept is at (2, 0). The slope of the tangent line there is undefined. The other x-intercept is at (0, 0) where the slope of the tangent line is zero.

2.(c) (i) The tangent line is horizontal at four points: $(1/\sqrt{2}, 1)$, $(1/\sqrt{2}, -1)$, $(-1/\sqrt{2}, 1)$ and $(-1/\sqrt{2}, -1)$.
(ii) The tangent line is vertical at two points: (–1. 0) and (1, 0).
(iii) The x-intercepts are (–1. 0), (1, 0) and (0, 0). At (–1. 0) and (1, 0), the slope of the tangent line is undefined. The graph crosses at (0, 0) twice, once with a slope of $m = 2$ and later with a slope of $m = -2$.

2.(d) (i) The tangent line is horizontal when $r = 1$ and $\theta = \pi/2$, when $r = 1$ and $\theta = 3\pi/2$, when $r = 2/3$ and $\theta = \sin^{-1}(1/\sqrt{6})$, when $r = 2/3$ and $\theta = \pi - \sin^{-1}(1/\sqrt{6})$, when $r = 2/3$ and $\theta = \pi + \sin^{-1}(1/\sqrt{6})$, and when $r = 2/3$ and $\theta = 2\pi - \sin^{-1}(1/\sqrt{6})$.
(ii) The tangent line is vertical when $r = 1$ and $\theta = 0$, when $r = 1$ and $\theta = \pi$, when $r = 2/3$ and $\theta = 3\pi/2 - \sin^{-1}(1/\sqrt{6})$, when $r = 2/3$ and $\theta = 3\pi/2 + \sin^{-1}(1/\sqrt{6})$, when $r = 2/3$ and $\theta = \pi/2 - \sin^{-1}(1/\sqrt{6})$, and when $r = 2/3$ and $\theta = \pi/2 + \sin^{-1}(1/\sqrt{6})$.

3.(a) The graph with the point P indicated is shown below.



3.(b) $x = \dfrac{3t}{1+t^3}$.

3.(c) $y = \dfrac{3t^2}{1+t^3}$.

3.(d) The interval is $0 \le t < \infty$.

4.(a) (i) $x = 2 + t$ $\quad y = 3 - t$ $\quad z = -4 - 2t$
(ii) $x - 2 = -y + 3 = (-z - 4)/2$

4.(b) (i) $x = 1$ $\quad y = 1$ $\quad z = 1 + t$
(ii) $x - 1 = y - 1 = 0$; z is arbitrary.

4.(c) (i) $x = 2 + 2t$ $\quad y = -3 - t$ $\quad z = 4 + 3t$
(ii) $(x - 2)/2 = -y - 3 = (z - 4)/3$

5.(a) 74/3.

5.(b) $\frac{\pi\sqrt{2}}{4}$.

5.(c) $\sqrt{5} \cdot \left(e^{2\pi} - 1\right)$

5.(d) $\pi\sqrt{1+4\pi^2}+\frac{1}{2}\ln\left(2\pi+\sqrt{1+4\pi^2}\right)$

5.(e) The integral for the arc length is the rather formidable:

$$\text{Arc length} = 6\cdot\int_0^1 \frac{\sqrt{1+4t^2-4t^3-4t^5+4t^6+t^8}}{\left(1+t^3\right)^2}\,dt.$$

Integration of this on a TI-83 Plus gives a value of about 4.91749 for the arc length.

6.(a) $2x - 7y + 17z = 78$.

6.(b) $3x + 2y + z = 6$.

6.(c) $7x - 5y - 2z = 9$.

7.(a) Area = $\dfrac{2\pi + 3\sqrt{3}}{6}$.

7.(b) Area = $\dfrac{5\pi - 6\sqrt{3}}{24}$.

7.(c) Area = $\dfrac{39\sqrt{3} - 10\pi}{6}$.

7.(d) Area = $\dfrac{2 - \sqrt{2}}{2}$.

8. The area is approximately 31271.643 square units.

9.(a) Each edge can be represented by a vector that is formed by subtracting one of the points away from another. The six vectors that can be so formed are:

$\langle 1, 1, 0\rangle$ $\langle 1, 0, 1\rangle$ $\langle 0, 1, 1\rangle$

$\langle 0, -1, 1\rangle$ $\langle -1, 0, 1\rangle$ $\langle -1, 1, 0\rangle$

All of these vectors have a length of $\sqrt{2}$.

9.(b) $\pi/3$ radians or 60°.

9.(c) About 109.5°.

10. The polar equation for the curve is $r = a + b\cdot\sin(\theta)$. Using $a = 1$ and $b = 2$ gives a curve that looks like the one shown below.