Unit Test 1 Review Problems – Set A

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. For each of the curves listed below, write equations for the curve in both Cartesian and polar coordinates.

(a) The vertical line that passes through the point (2, 0).

(b) The line with slope −1 that passes through the point (2, −1).

(c) The circle with center located at (0, −4) that passes through the origin.

(d) The circle with radius 5 and center located at the point (3, 4).

2. For each of the curves given below, find:

(i) The points on the curve where the tangent line is horizontal, and,

(ii) The points on the curve where the tangent line is vertical, and,

(iii) The slope of the tangent line at any point where the curve intersects the x-axis.

(a) $x = t^2$ and $y = t^3 - 3t$

(b) $r = 1 + \cos(\theta)$

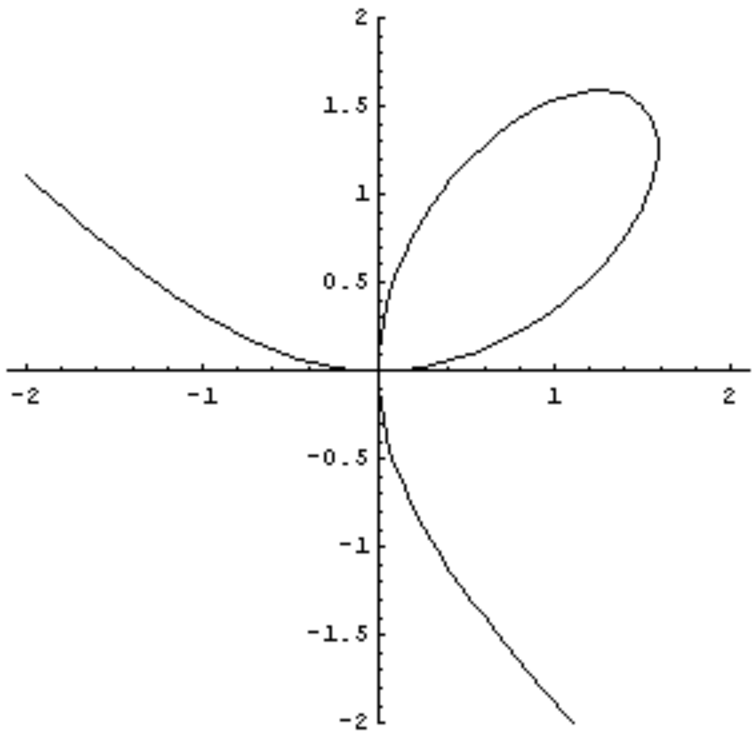
(c) $x = \sin(t)$ and $y = \sin(2t)$

(d) $r = \cos(2\theta)$. (Don't worry about (iii) for this one – (i) and (ii) are enough.)

3. The folium of Descartes is the curve in the xy-plane with the Cartesian equation:

$$x^3 + y^3 = 3xy.$$

A plot of this folium is shown below.



The point of this problem is to find a pair of parametric equations to represent the loop of the folium that lies in the first quadrant of the xy-plane.

(a) Consider a number $t > 0$. On the diagram given above draw the point, P, representing the intersection of the line $y = t{\cdot}x$ with the folium.

(b) Solve for the x coordinate of P in terms of t (but not y).

(c) Solve for the y coordinate of P in terms of t (but not x).

(d) Find the interval of t-values that will complete the loop of the folium.

4. In each of the following cases, write down:

(i) parametric equations, and,

(ii) symmetric equations

for the straight line (in three-dimensional space) that is described.

(a) The line passes through $(2, 3, -4)$ with direction $\langle 1, -1, -2 \rangle$.

(b) The line passes through $(1, 1, 1)$ and is perpendicular to the xy-plane.

(c) The line passes through $(2, -3, 4)$ and is perpendicular to the plane with equation $2x - y + 3z = 4$.

5. For each of the curves given below, find the length of the curve. Except where otherwise indicated, you should do all integrals by hand (and not on your calculator).

(a) $x = 2t$ and $y = \frac{2}{3}t^{\frac{3}{2}}$ with $5 \leq t \leq 12$.

(b) $x = \sin(t) - \cos(t)$ and $y = \sin(t) + \cos(t)$ with $\pi/4 \leq t \leq \pi/2$.

(c) $r = e^{\frac{\theta}{2}}$ with $0 \leq \theta \leq 4\pi$.

(d) $x = t{\cdot}\cos(t)$ and $y = t{\cdot}\sin(t)$ with $0 \leq t \leq 2\pi$.

(e) (Calculator okay on this part.) The portion of $x^3 + y^3 = 3xy$ that lies within the first quadrant of the xy-plane.

6. In each part of this problem, a plane (that resides in three-dimensional space) is described. For each description, find an equation for the plane that is described.

(a) The plane contains the point (2, 4, 6) and the line whose parametric equations are:

$$x = 7 - 3t \qquad y = 3 + 4t \qquad z = 5 + 2t.$$

(b) The plane contains the point (1, 1, 1) and intersects the xy-plane in the same line as the plane $3x + 2y - z = 6$.

(c) The plane contains the points (1, 0, −1) and (2, 1, 0). It is parallel to the line of intersection between the planes $x + y + z = 5$ and $3x - y = 4$.

7. In each case find the area of the region described. It may be helpful to graph the curves, and be certain to pay careful attention to words like "inside" and "outside" in the description of each region.

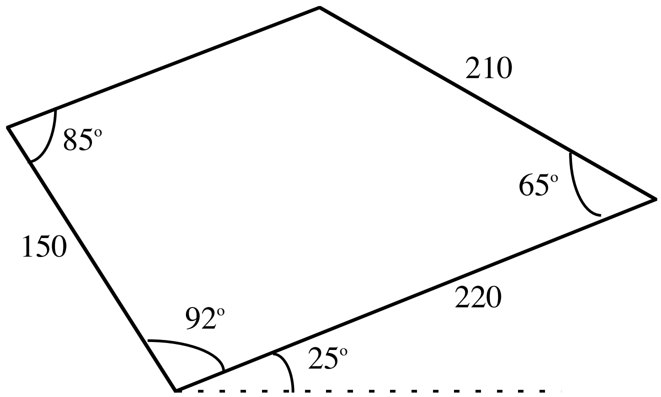
(a) Inside $r = 2{\cdot}\sin(\theta)$ and outside $r = 1$.

(b) Inside both of the curves $r = \cos(\theta)$ and $r = \sqrt{3}{\cdot}\sin(\theta)$.

(c) Inside $r = 3 + 2{\cdot}\cos(\theta)$ and outside $r = 4$.

(d) Inside both of the curves $r^2 = \cos(2\theta)$ and $r^2 = \sin(2\theta)$.

8. Find the area of the irregular quadrilateral shown in the diagram given below.

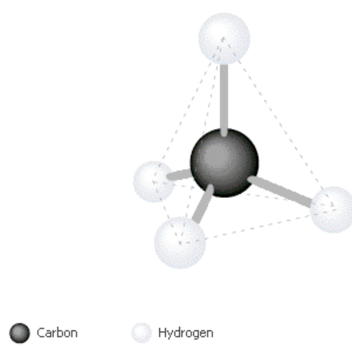


(Hint: Try dividing the area into triangles, then remember that the area of the triangle formed by two vectors **a** and **b** is $0.5 \cdot |\mathbf{a} \times \mathbf{b}|$.)

9. **(a)** Prove that the points (0, 0, 0), (1, 1, 0), (1, 0, 1) and (0, 1, 1) are the vertices (points) of a regular tetrahedron by showing that each of the six edges has a length of $\sqrt{2}$.

(b) Find the angle between any two edges of the tetrahedron.

(c) The molecular structure of methane (CH_4) features a hydrogen atom at each of the four vertices of a regular tetrahedron with the carbon atom at its center (see diagram). Suppose the axes and scales have been chosen so that the tetrahedron is the same one as in Part (a), whose center is located at the point (0.5, 0.5, 0.5). Find the *bond angle* between adjacent carbon-hydrogen bonds.



10. A family of curves is defined in the *xy*-plane by the equation:

$$a^2 \cdot (x^2 + y^2) = (x^2 + y^2 - b \cdot y)^2.$$

Sketch a graph of the curve when $a = 1$ and $b = 2$. (Hint: is there an easier coordinate system than Cartesian coordinates?)