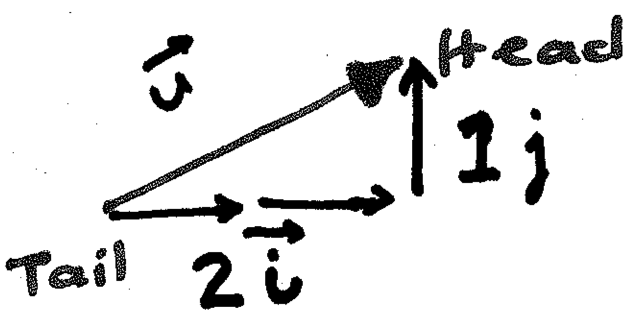
Outline

1. Vectors and coordinates.
2. Physics and tension.
3. Dot product.

1. Vectors and Coordinates

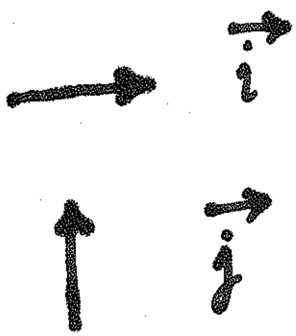
GRAPHICAL | **ALGEBRAIC**

Representation



$$\boxed{\vec{u} = \langle u_1, u_2 \rangle}$$

$$= \langle u_1, 0 \rangle + \langle 0, u_2 \rangle$$



$$= u_1 \langle 1, 0 \rangle + u_2 \langle 0, 1 \rangle$$

$\vec{u} = 2\vec{i} + 1 \cdot \vec{j}$

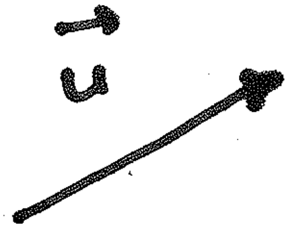
$$\vec{i} = \langle 1, 0 \rangle$$

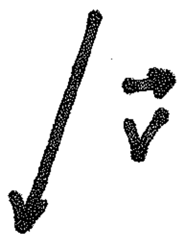
$$\vec{j} = \langle 0, 1 \rangle$$

$$\boxed{\vec{u} = u_1 \cdot \vec{i} + u_2 \cdot \vec{j}}$$

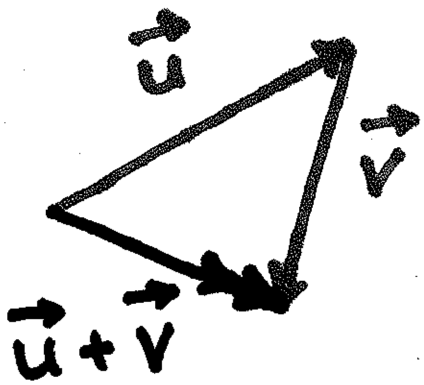
GRAPHICAL | ALGEBRAIC.

Adding and Subtracting





Put vectors together head to tail & draw resultant.



components of vector

$$\vec{u} = \langle u_1 , u_2 \rangle$$

$$\vec{u} = u_1 \cdot \vec{i} + u_2 \cdot \vec{j}$$

$$\vec{v} = \langle v_1 , v_2 \rangle$$

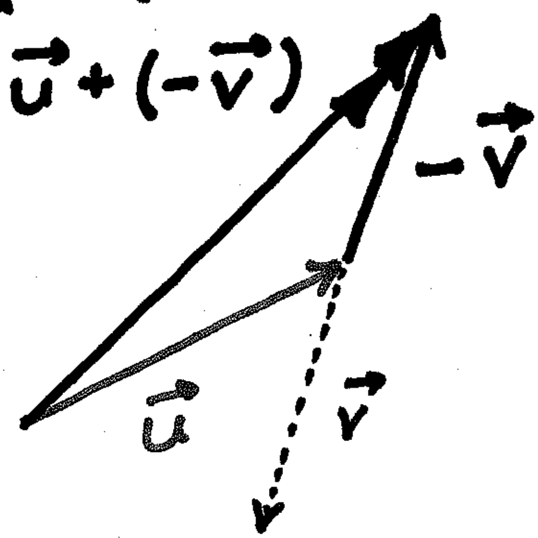
$$= v_1 \vec{i} + \vec{v}_2 \cdot \vec{j}$$

$$\vec{u} + \vec{v} = \langle u_1 + v_1 , u_2 + v_2 \rangle$$

$$\vec{u} + \vec{v} = (u_1 + v_1) \cdot \vec{i} + (u_2 + v_2) \cdot \vec{j}$$

Subtract: Add the negative of the vector.

$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$



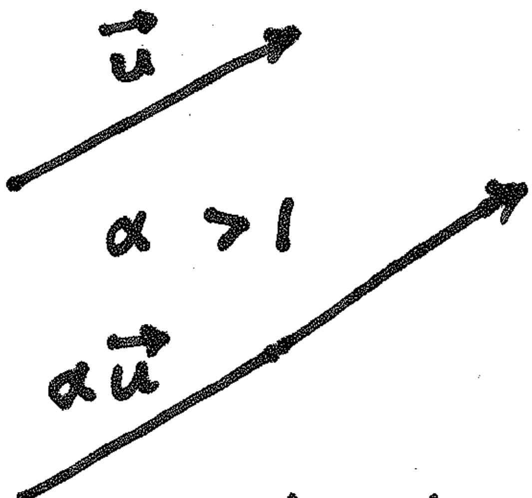
$$-\vec{v} = \langle -v_1, -v_2 \rangle$$

$$\vec{u} - \vec{v} = \langle u_1, u_2 \rangle$$
$$+ \langle -v_1, -v_2 \rangle$$
$$= \langle u_1 - v_1, u_2 - v_2 \rangle$$
$$= (u_1 - v_1)\vec{i} + (u_2 - v_2)\vec{j}$$

Multiplying by a number (salar)

α = number

$$\alpha \vec{v} = \langle \alpha v_1, \alpha v_2 \rangle$$
$$= (\alpha v_1)\vec{i} + (\alpha v_2)\vec{j}$$



same direction
different length.

Length of a Vector

$$\vec{v} = \langle v_1, v_2 \rangle = v_1 \vec{i} + v_2 \vec{j}$$

Length of 2D vector: $|\vec{v}| = \sqrt{v_1^2 + v_2^2}$

abs. value signs
mean length or
magnitude of vector.

$$\vec{v} = \langle v_1, v_2, v_3 \rangle = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$$

Length of 3D Vector: $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

where:

$\vec{i} = \langle 1, 0, 0 \rangle$ x-axis

$\vec{j} = \langle 0, 1, 0 \rangle$ y-axis

$\vec{k} = \langle 0, 0, 1 \rangle$ z-axis.

- A <u>unit vector</u> is a vector with length 1.

<u>Example</u>

$\vec{v} = \langle 1, 2, 10 \rangle$. Find a unit vector $\vec{u}$ that has the same direction as $\vec{v}$.

<u>Solution</u>

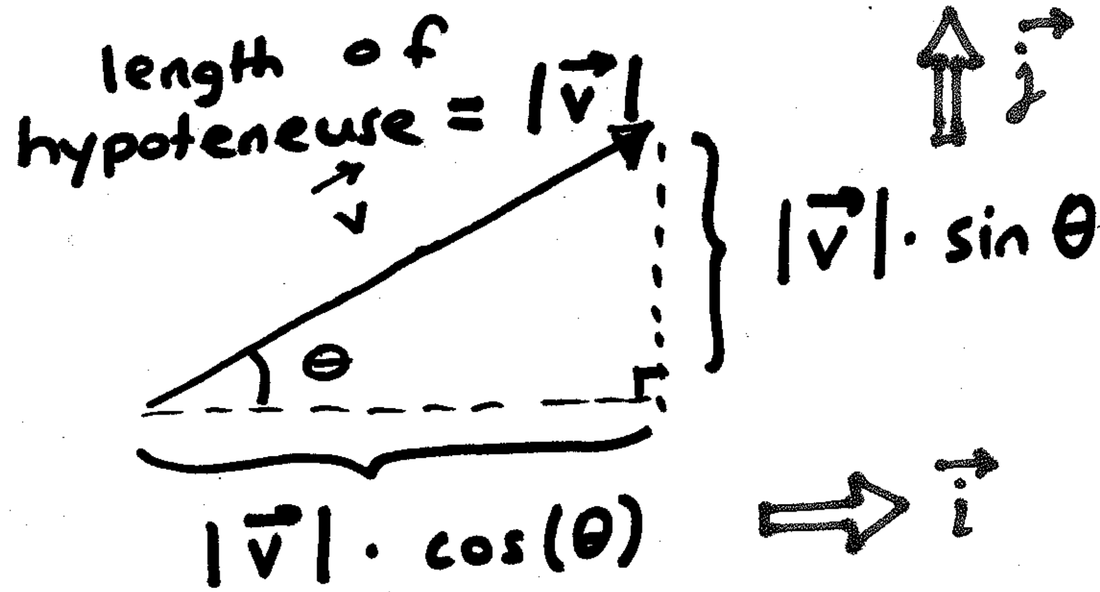
$$|\vec{v}| = \sqrt{1^2 + 2^2 + 10^2} = \sqrt{105}$$

$$\vec{u} = \frac{1}{|\vec{v}|} \cdot \vec{v}$$

$$= \left\langle \frac{1}{\sqrt{105}}, \frac{2}{\sqrt{105}}, \frac{10}{\sqrt{105}} \right\rangle.$$

2. Physics and Tension

①



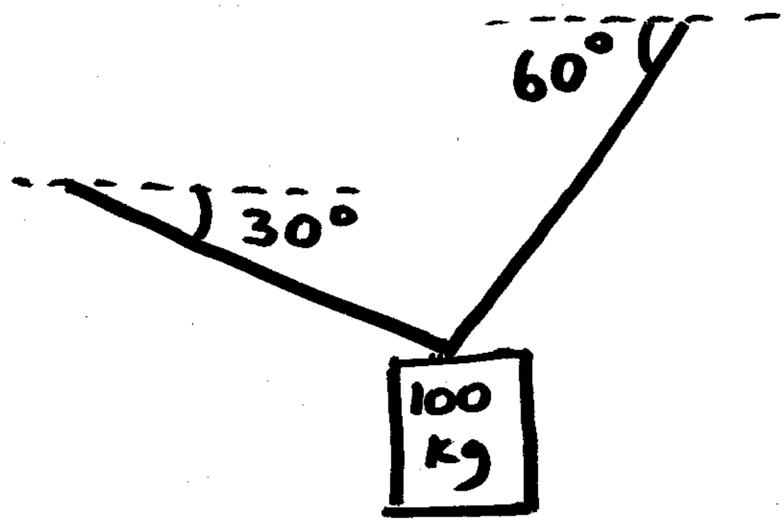
$$\vec{v} = |\vec{v}| \cdot \cos(\theta) \cdot \vec{i} + |\vec{v}| \cdot \sin(\theta) \cdot \vec{j}$$

② In a static situation:

- Horizontal forces sum to zero
- Vertical forces sum to zero.

Example

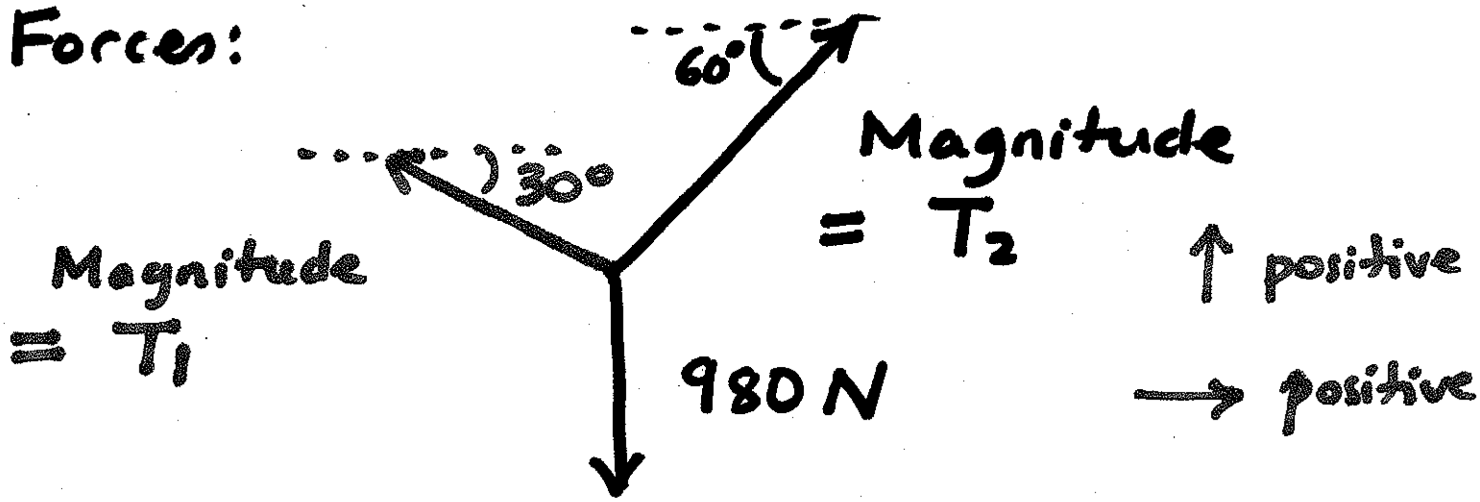
A man of 100 kg is suspended from a rope. The breaking strain of the rope is 900[redacted] N.



Will either rope break?

Solution

Forces:



Force vector:

$\theta = 30°$

$$\vec{T_1} = -T_1 \cdot \cos(\theta)\,\vec{i} + T_1 \cdot \sin(30°)\,\vec{j}$$

$$\vec{T_2} = T_2 \cdot \cos(60°)\,\vec{i} + T_2 \cdot \sin(60°)\,\vec{j}$$

$$\text{gravity} = 0 \cdot \vec{i} - 980\,\vec{j}$$

Horizontal:

$$-T_1 \cos(\overset{30^\circ}{\theta})\,\vec{i} + T_2 \cdot \cos(60^\circ)\,\vec{i} = 0\,\vec{i}$$

Vertical

$$T_1 \sin(30^\circ)\,\vec{j} + T_2 \sin(60^\circ)\,\vec{j} - 980\,\vec{j} = 0\,\vec{j}$$