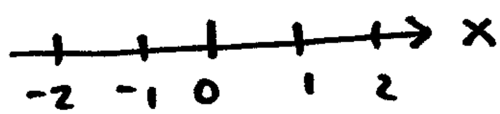
Outline

1. Planes and spheres in 3D.
2. Vectors, unit vectors and coordinates.
3. Tension.

1. Planes, Spheres, Lines in 3D

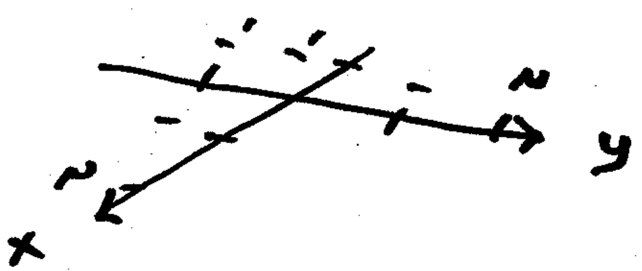
Notation

$\mathbb{R}$ - one dimensional real line



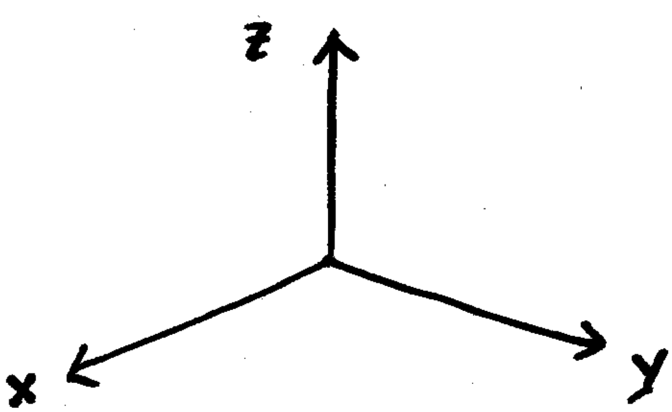
x

$\mathbb{R}^2$ - two dimensional flat plane



(x, y)

$\mathbb{R}^3$ - three dimensional space.



(x, y, z)

Example

What does $x = 2$ refer to in:

(a) $\mathbb{R}$ (b) $\mathbb{R}^2$ (c) $\mathbb{R}^3$?

What does $x = 2$, $y = 3$ refer to in (d) $\mathbb{R}^2$ (e) $\mathbb{R}^3$?

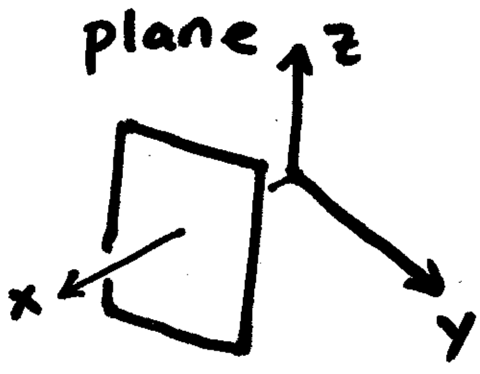
Solution

- Variables that aren't mentioned can have any possible value.

(a) $\mathbb{R}$: $\left. x = 2 \right\}$ a point.

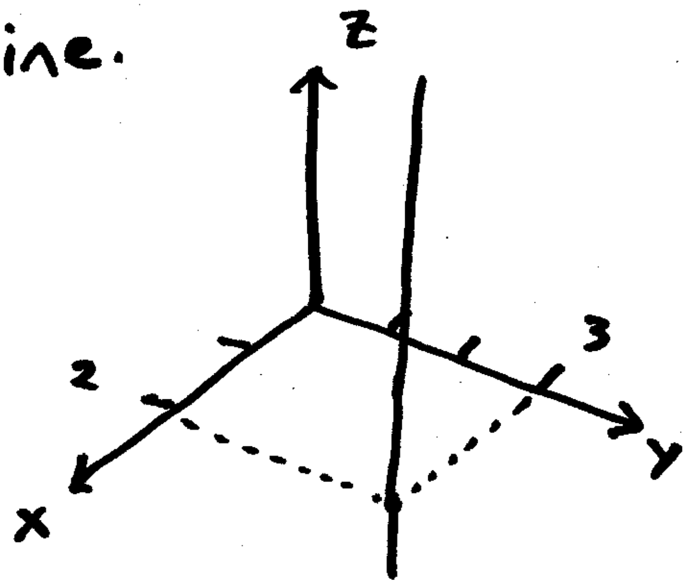
(b) $\mathbb{R}^2$: $\left.\begin{array}{l} x = 2 \\ y = \text{any value} \end{array}\right\}$ vertical line

(c) $\mathbb{R}^3$: $\left.\begin{array}{l} x = 2 \\ y = \text{any value} \\ z = \text{any value} \end{array}\right\}$ plane



(d) $\left.\begin{array}{l} x = 2 \\ y = 3 \end{array}\right\}$ point

(e) $\left.\begin{array}{l} x = 2 \\ y = 3 \\ z = \text{any value} \end{array}\right\}$ line.



Equations of Lines, Planes, Spheres

(a) Lines

Case 1: Have two points:

$(x_0, y_0, z_0) \quad (x_1, y_1, z_1)$

and want line that joins them.

$$x(t) = (1-t)\cdot x_0 + t\cdot x_1$$

$$y(t) = (1-t)\cdot y_0 + t\cdot y_1$$

$$z(t) = (1-t)\cdot z_0 + t\cdot z_1$$

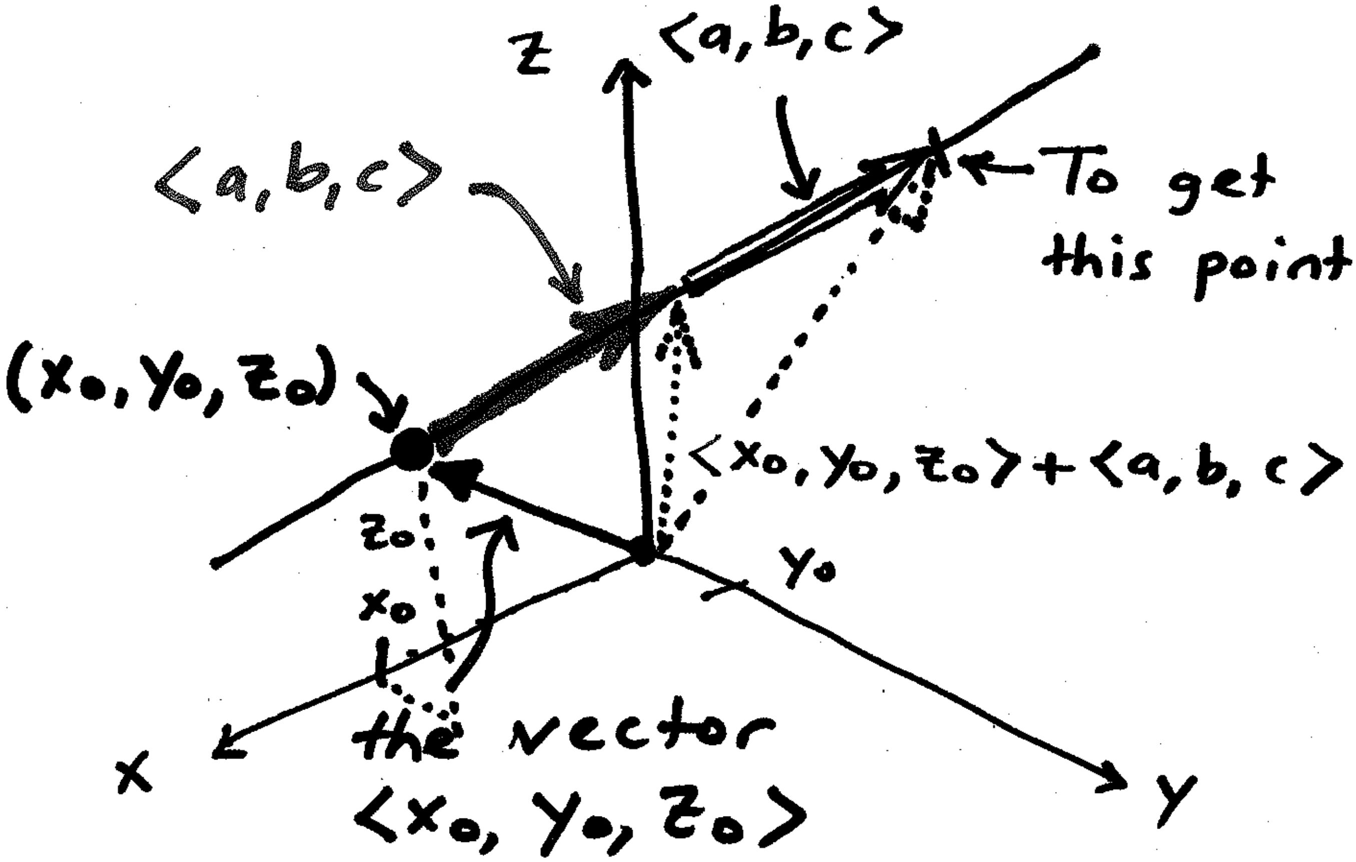
- Variable is: t.

<u>Case 2:</u> Have one point (x_0, y_0, z_0) on line and a vector $\langle a, b, c \rangle$ that gives the direction of the line.

these mean a vector.

$$\langle x(t), y(t), z(t)\rangle = \langle x_0, y_0, z_0\rangle + t\cdot\langle a, b, c\rangle.$$

Why does this work?



The coordinates of the point on the line we're looking for are:

$$\langle x_0, y_0, z_0 \rangle + \langle a, b, c \rangle + \langle a, b, c \rangle$$

$$= \langle x_0, y_0, z_0 \rangle + \underset{\boxed{t = 2}}{t} \cdot \langle a, b, c \rangle$$

(b) Equation of a Plane

"Normal" Equation

Have a point (x_0, y_0, z_0) on the plane and a vector (normal vector)

arrow on top means this is a vector

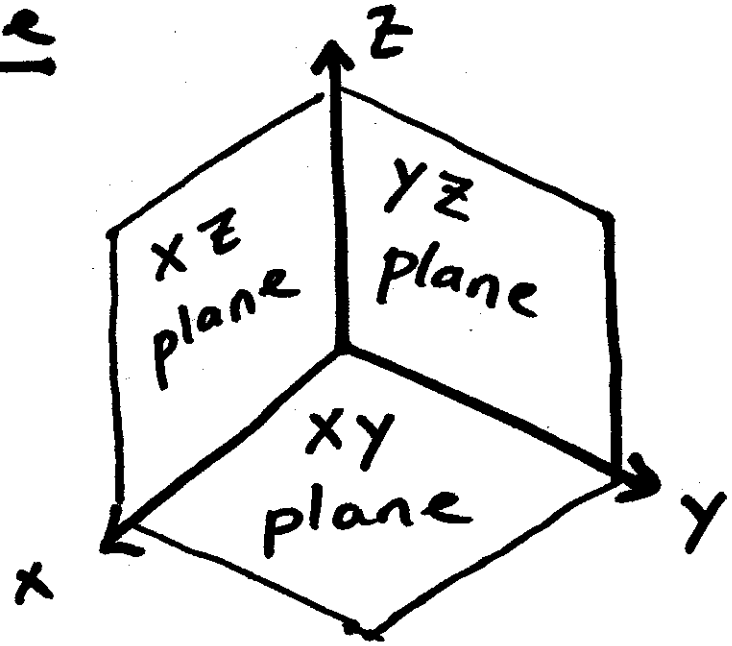
$$\vec{n} = \langle n_1, n_2, n_3 \rangle.$$

Normal vector means that it is perpendicular to the surface of the plane.

Equation for plane is:

$$\boxed{n_1 \cdot (x - x_0) + n_2 \cdot (y - y_0) + n_3 \cdot (z - z_0) = 0}$$

Example



Find normal equations for the 3 planes.

Solution

(a) xy-plane $(x_0, y_0, z_0) = (2, 3, 0)$.

$$\vec{n} = \langle 0, 0, 1 \rangle$$

$$0 \cdot (x-2) + 0 \cdot (y-3) + 1 \cdot (z-0) = 0$$

$$\boxed{z = 0}$$

(b) xz-plane $(x_0, y_0, z_0) = (1, 0, 2)$

$$\vec{n} = \langle 0, 2, 0 \rangle$$

$$0\cdot(x-1) + 2(y-0) + 0(z-2) = 0$$

$$\boxed{y = 0}$$

(c) $\boxed{x = 0}$.

(c) <u>Spheres</u>

<u>Distance formula in 3D</u>

Distance from (x_0, y_0, z_0) to (x_1, y_1, z_1) is:

$$\rho = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}$$

<u>Sphere in 3D</u>

Center at (x_0, y_0, z_0) and radius ρ:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = \rho^2$$

Example

Find the distance from the point $(2, 1, 7)$ to:

(a) The x-z plane.

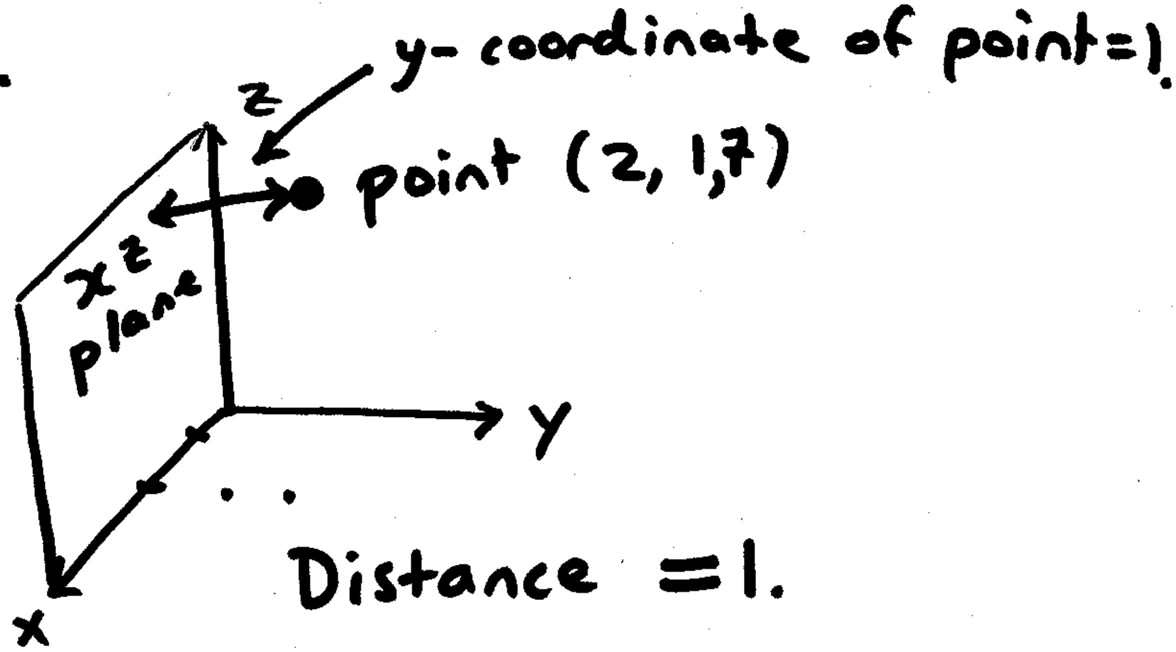
(b) The y-axis.

(c) The plane:

$$2(x-3) + 3(y-1) - 2(z-2) = 0.$$

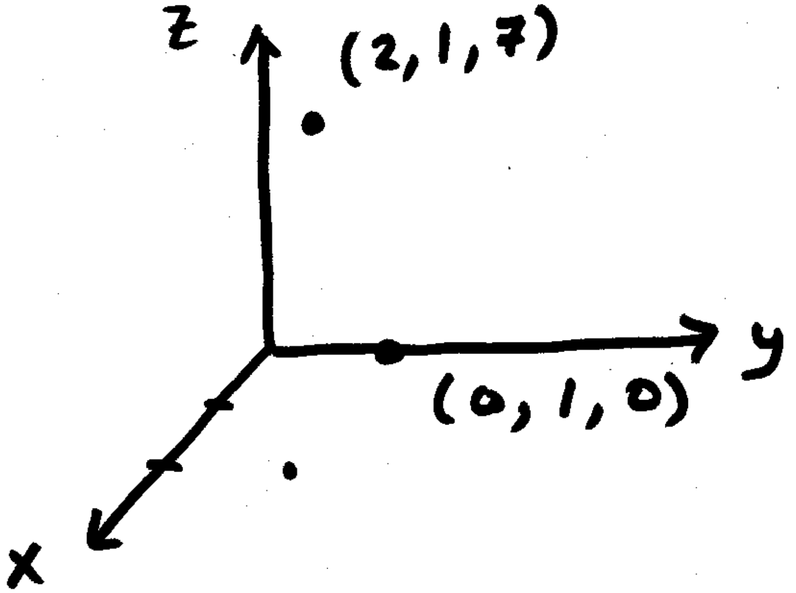
Solution

(a)



Distance $= 1$.

(b)



Distance $= \sqrt{2^2 + 0^2 + 7^2} = \sqrt{53}$.