

Outline

1. Polar equations for conic sections.
2. Sketching polar equations.
3. Planes and spheres in 3D.

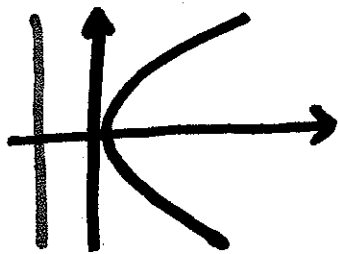
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Quiz tomorrow.

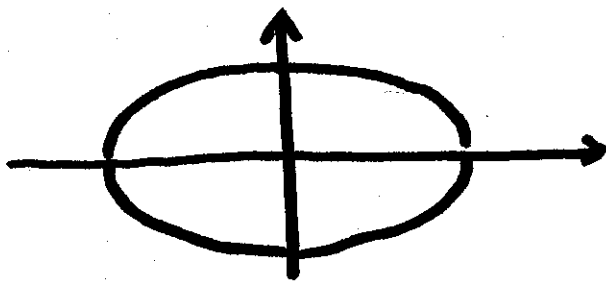
1. Polar Equations for Conic Sections

Vertical Directrix

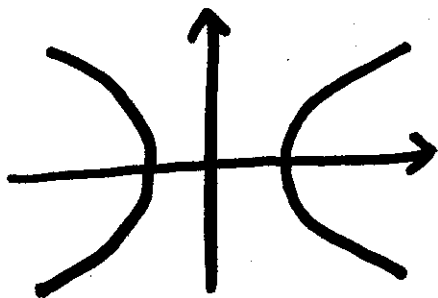
$$x = d$$



Parabola



Ellipse



Hyperbola.

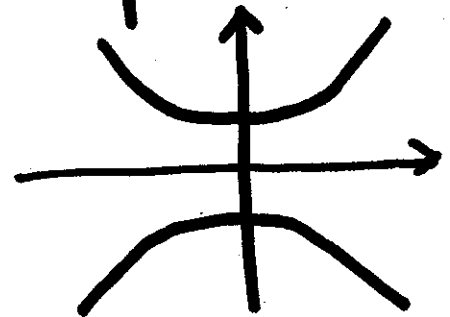
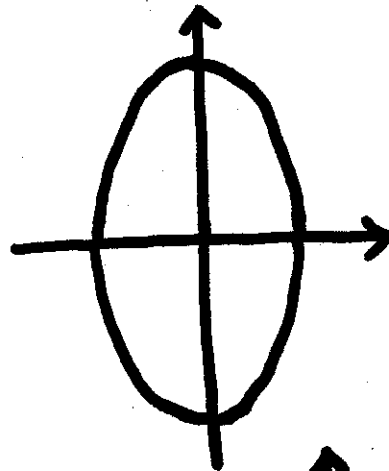
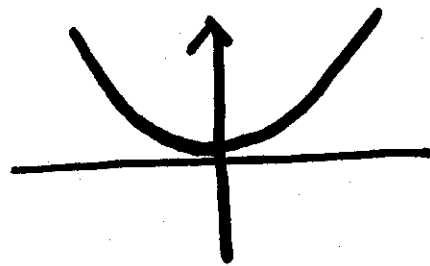
$$r = \frac{e \cdot d}{1 \pm e \cdot \cos \theta}$$

+ in denom $\Rightarrow x = +d$

- in denom $\Rightarrow x = -d$

Horizontal Directrix

$$y = d$$



$$r = \frac{e \cdot d}{1 \pm e \cdot \sin \theta}$$

+ in denom $\Rightarrow y = +d$

- in denom $\Rightarrow y = -d$

where: d is the x - or y -coordinate of the directrix.

e is the eccentricity.

Eccentricity e	Conic Section
0	Circle
$0 < e < 1$	Ellipse
1	Parabola
$e > 1$	Hyperbola.

2. Sketching Conic Sections Defined by Polar Equations

Example

$$r = \frac{6}{3 + 2 \cdot \sin \theta}$$

- want:
- (a) Eccentricity
 - (b) Directrix
 - (c) Type of curve
 - (d) Sketch in xy-plane.

Solution

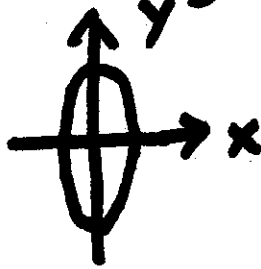
$$\begin{aligned} r &= \frac{6}{3 + 2 \cdot \sin \theta} \\ &= \frac{2}{1 + \frac{2}{3} \cdot \sin(\theta)} \\ &= \frac{3 \left(\frac{2}{3} \right)}{1 + \frac{2}{3} \cdot \sin(\theta)} \end{aligned}$$

(a) Eccentricity = $\frac{2}{3}$.

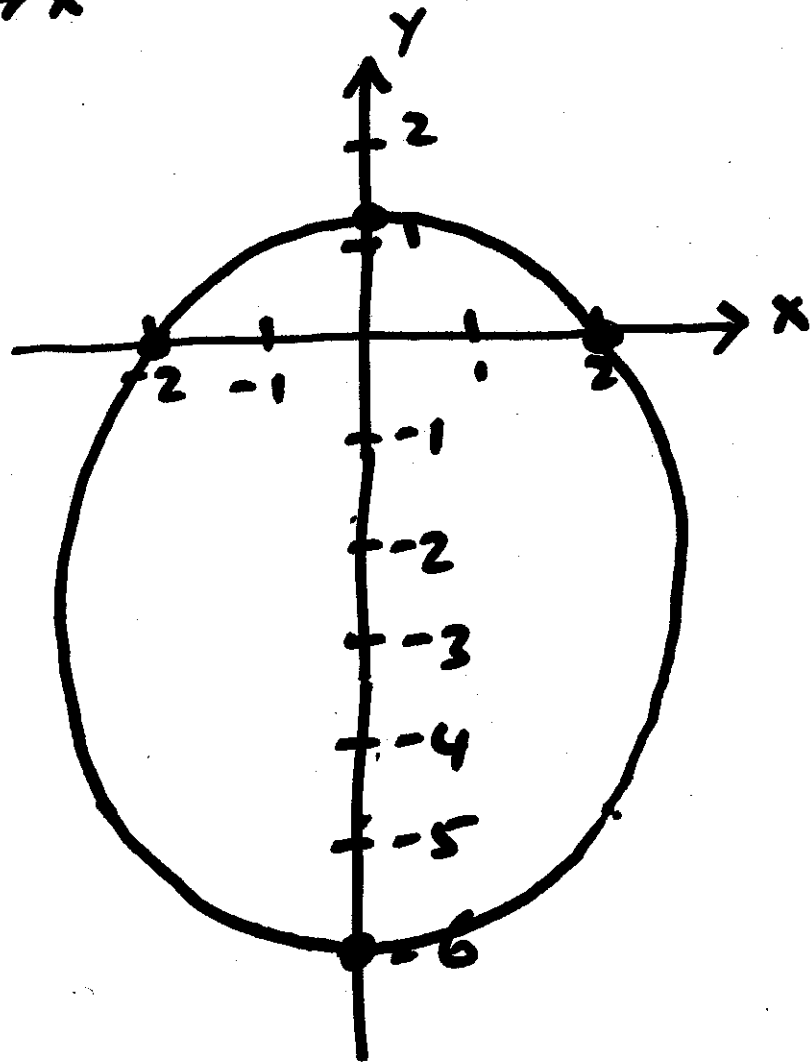
(b) Directrix: $y = 3$.

(c) Ellipse.

(d) Know its going to look like:



θ	r	x	y
0	2	2	0
$\pi/2$	$6/5$	0	$6/5$
π	2	-2	0
$3\pi/2$	6	0	-6



Example

Same requirements as before,
now with:

$$r = \frac{4}{2 - 3 \cdot \cos \theta}.$$

Solution

$$r = \frac{2}{1 - \frac{3}{2} \cos \theta}$$

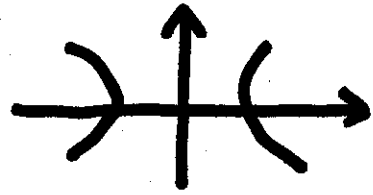
$$= \frac{\frac{3}{2} \cdot \frac{4}{3}}{1 - \frac{3}{2} \cdot \cos \theta}$$

(a) Eccentricity $e = 3/2$

(b) Directrix $x = -4/3.$

(c) Hyperbola ($e > 1$)

(d) Know its going to look something like:

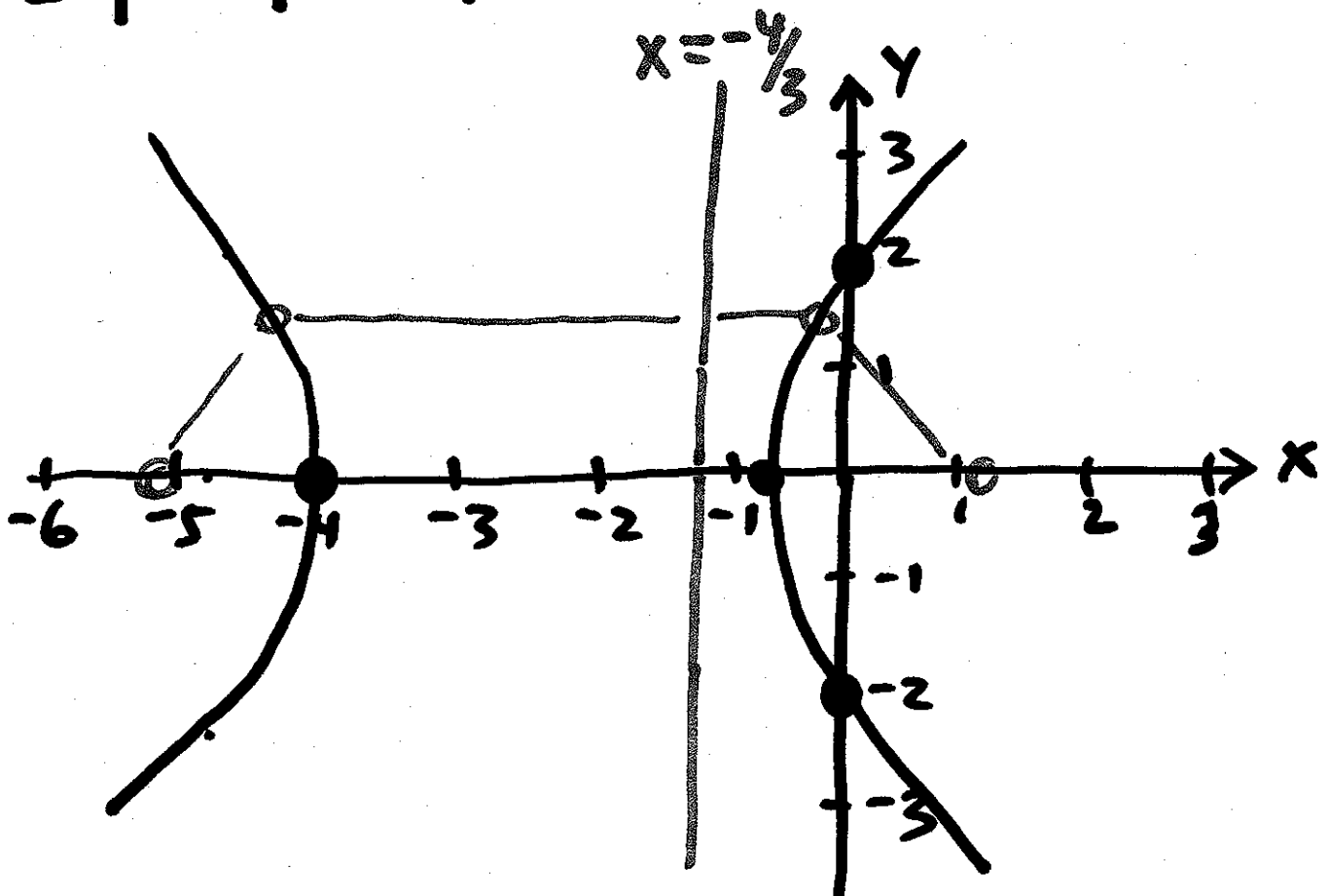


θ	r	x	y
0	-4	-4	0
$\pi/2$	2	0	2
π	$4/5$	$-4/5$	0
$3\pi/2$	2	0	-2

Remember:

$r = -4$ with $\theta = 0$ means plot point with

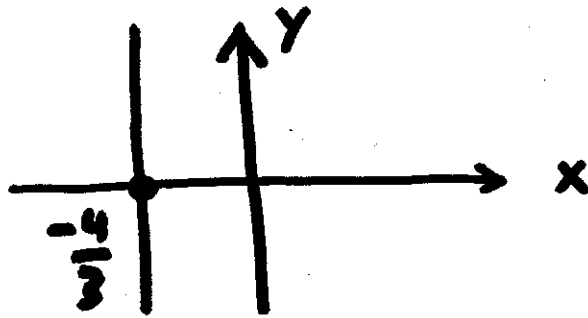
$r = 4$ $\theta = \pi$.



3. Planes and Spheres in 3D

1 Dimension: $x = -\frac{4}{3}$ point.

2 Dimensions: $x = -\frac{4}{3}$ line.



3 Dimensions: $x = -\frac{4}{3}$ plane.

