Outline

1. Conics in Cartesian Coordinates.

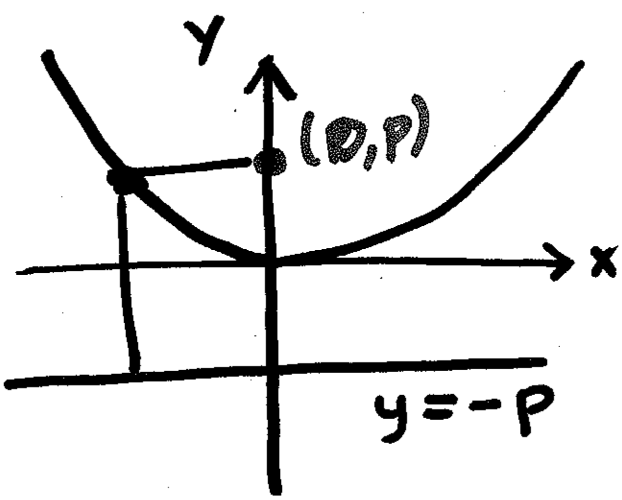
2. Conics in Polar Coordinates.

3. Sketching Polar Conics.

1. Conic Sections in Cartesian Coordinates

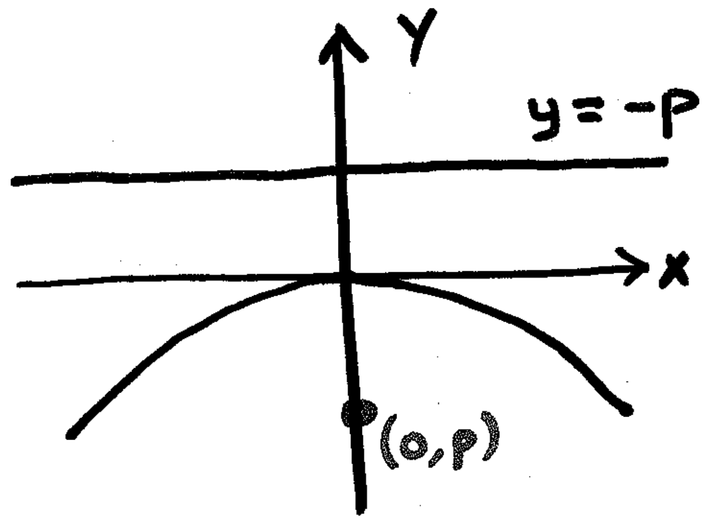
(a) Parabolas

- Fixed point (focus, F) and fixed line (directrix, ℓ) the points that are equidistant from both form a parabola.
- Four basic equations for a parabola:



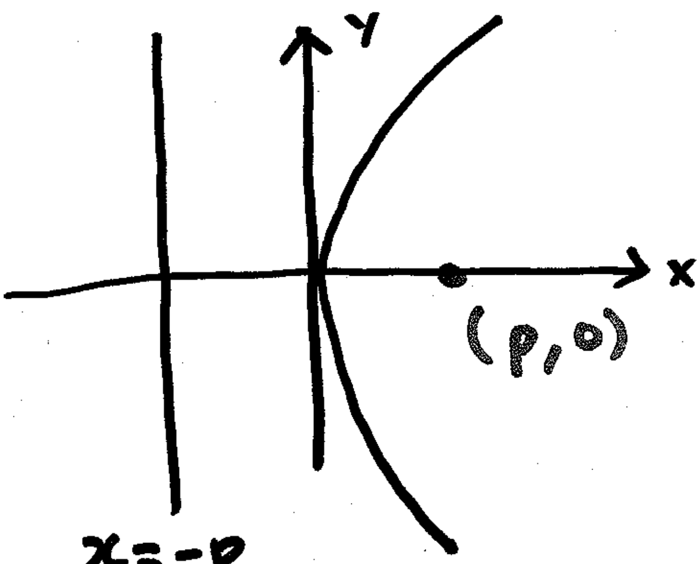
- Directrix parallel to x-axis
- $p > 0$

$$x^2 = 4py$$



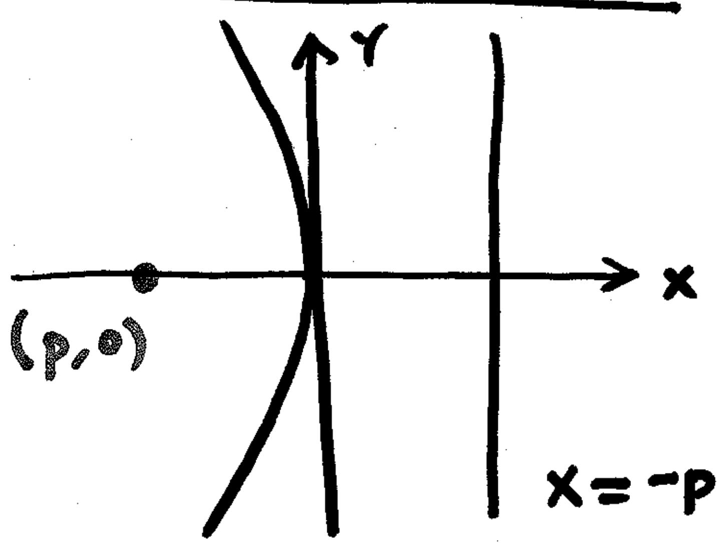
- Directric parallel to x-axis.
- $p < 0$

$$x^2 = 4py$$



- Directrix parallel to y-axis
- $p > 0$

$$y^2 = 4px$$

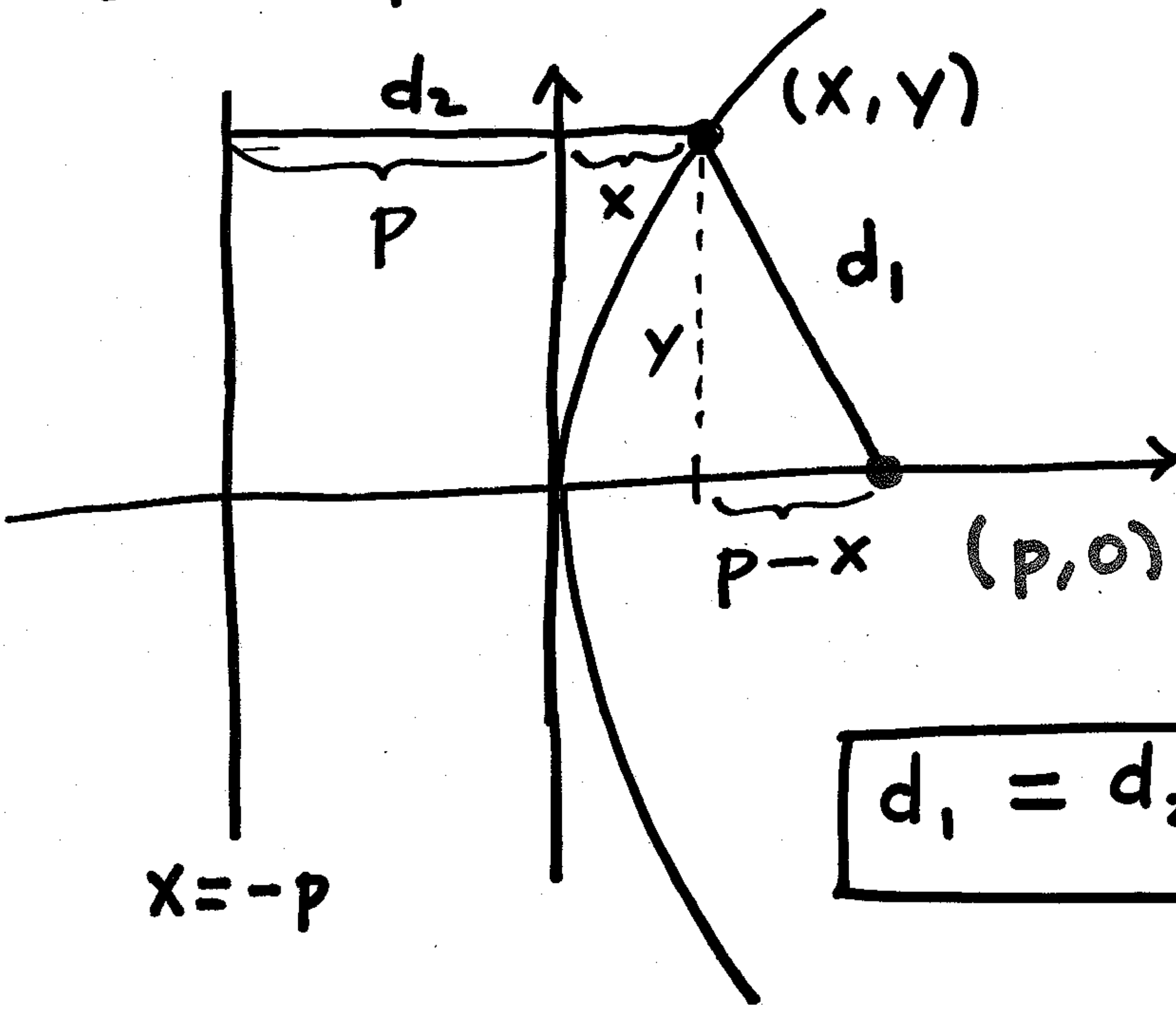


- Directrix parallel to y-axis.
- $p < 0$.

$$y^2 = 4px$$

Why $y^2 = 4px$?

(Assume $p > 0$).



$$\boxed{d_1 = d_2}$$

$$d_2 = p + x$$

$$d_1 = \sqrt{(p-x)^2 + y^2}$$

$$d_1^2 = d_2^2 : \quad (p-x)^2 + y^2 = (p+x)^2$$

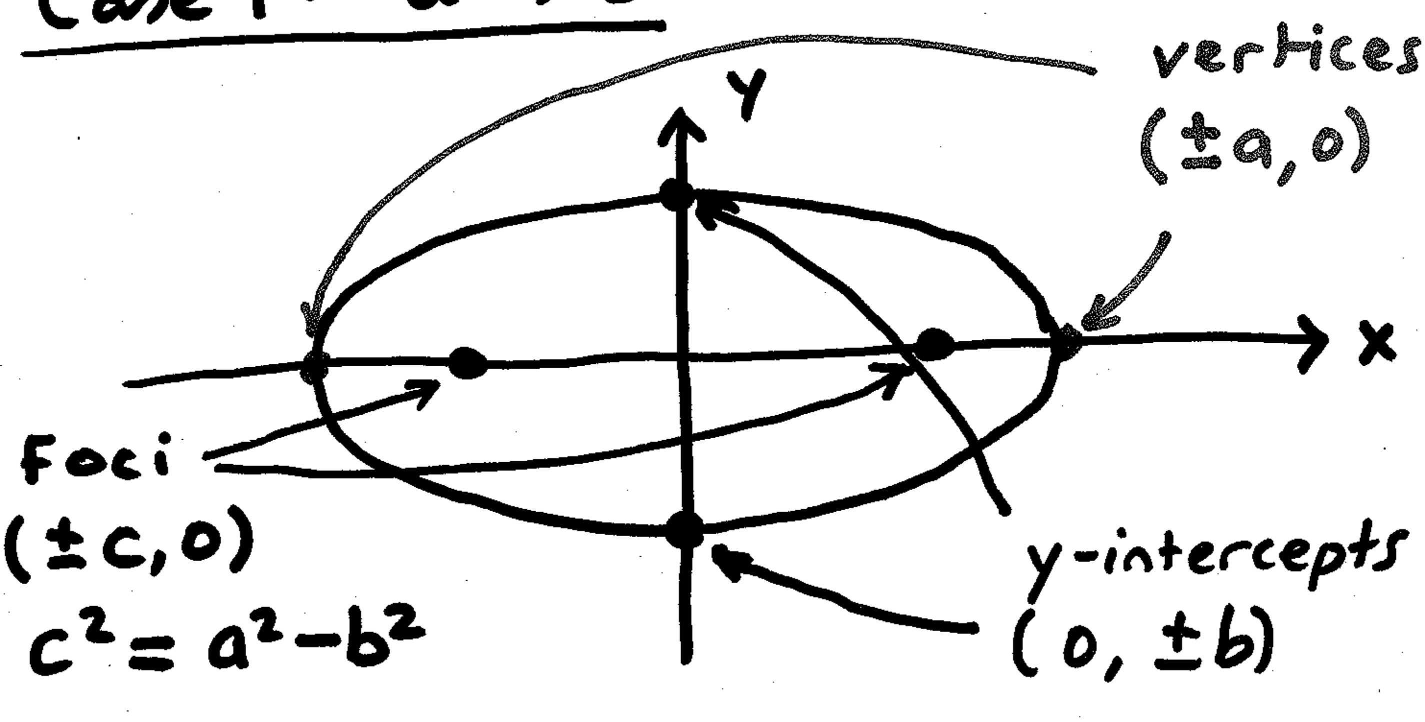
$$\cancel{p^2} - 2px + \cancel{x^2} + y^2 = \cancel{p^2} + 2px + \cancel{x^2}$$

$$\boxed{y^2 = 4px}$$

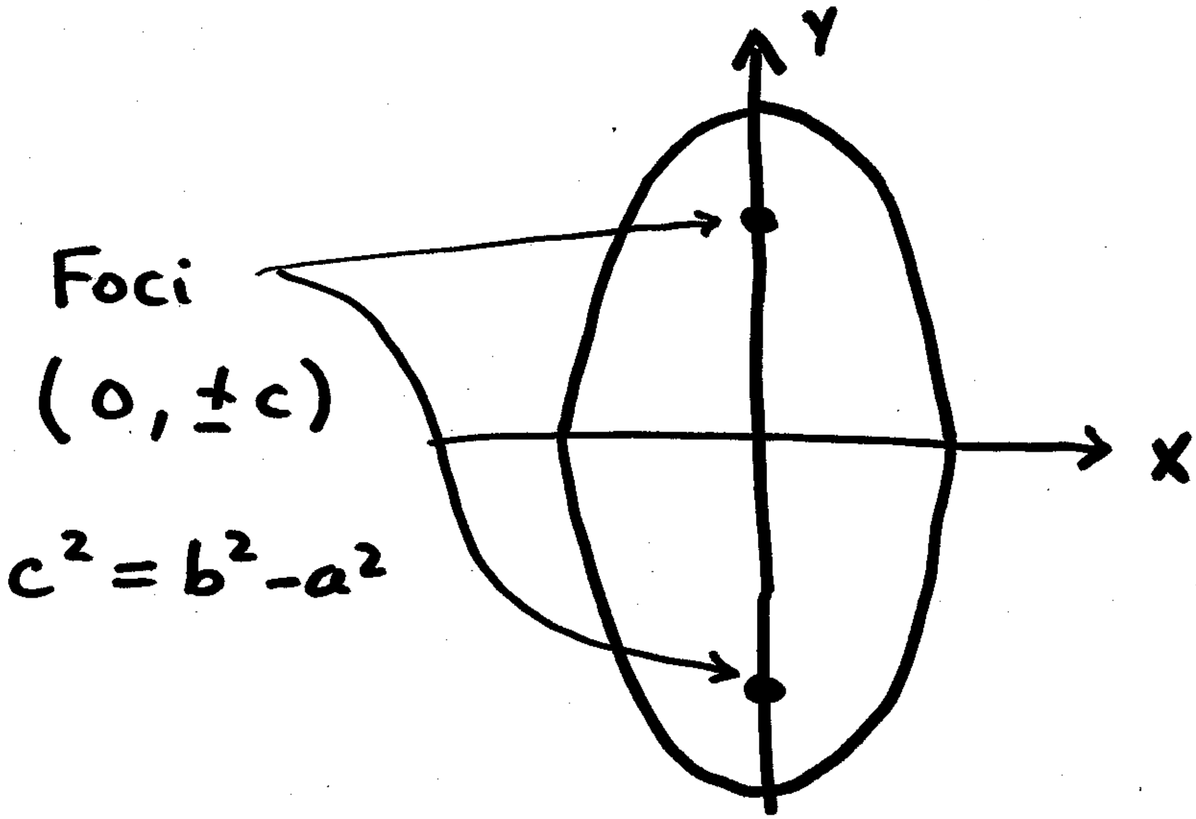
(b) Ellipses.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Case 1: $a^2 > b^2$



Case 2: $b^2 > a^2$



(c) Hyperbolas

- Difference in distances [redacted] from two fixed points (foci) is constant.

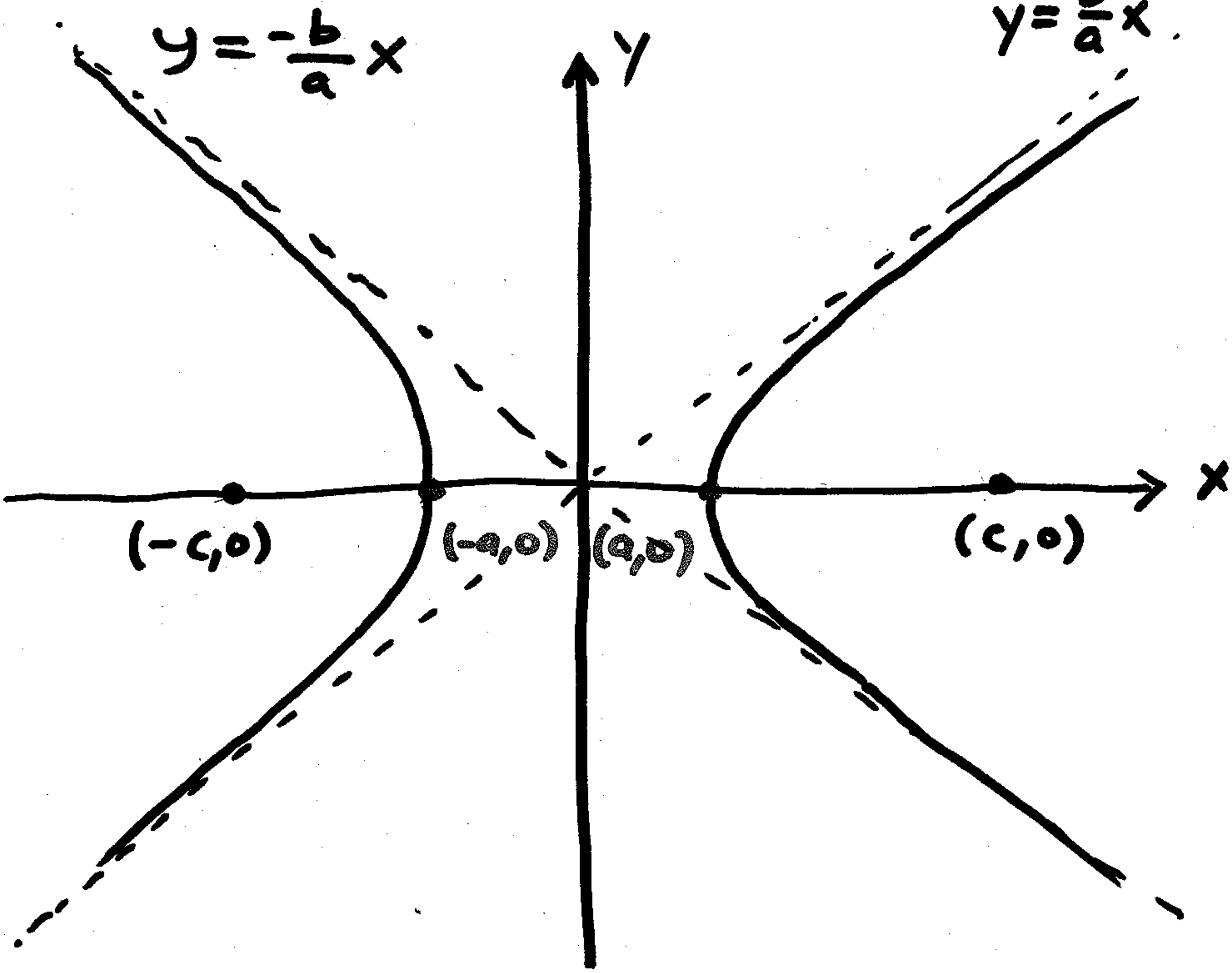
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Foci: Located at

$(\pm c, 0)$ where $c^2 = a^2 + b^2$

Vertices: $(\pm a, 0)$

Asymptotes: $y = \pm \frac{b}{a} \cdot x$



2. Conic Sections in Polar Coordinates

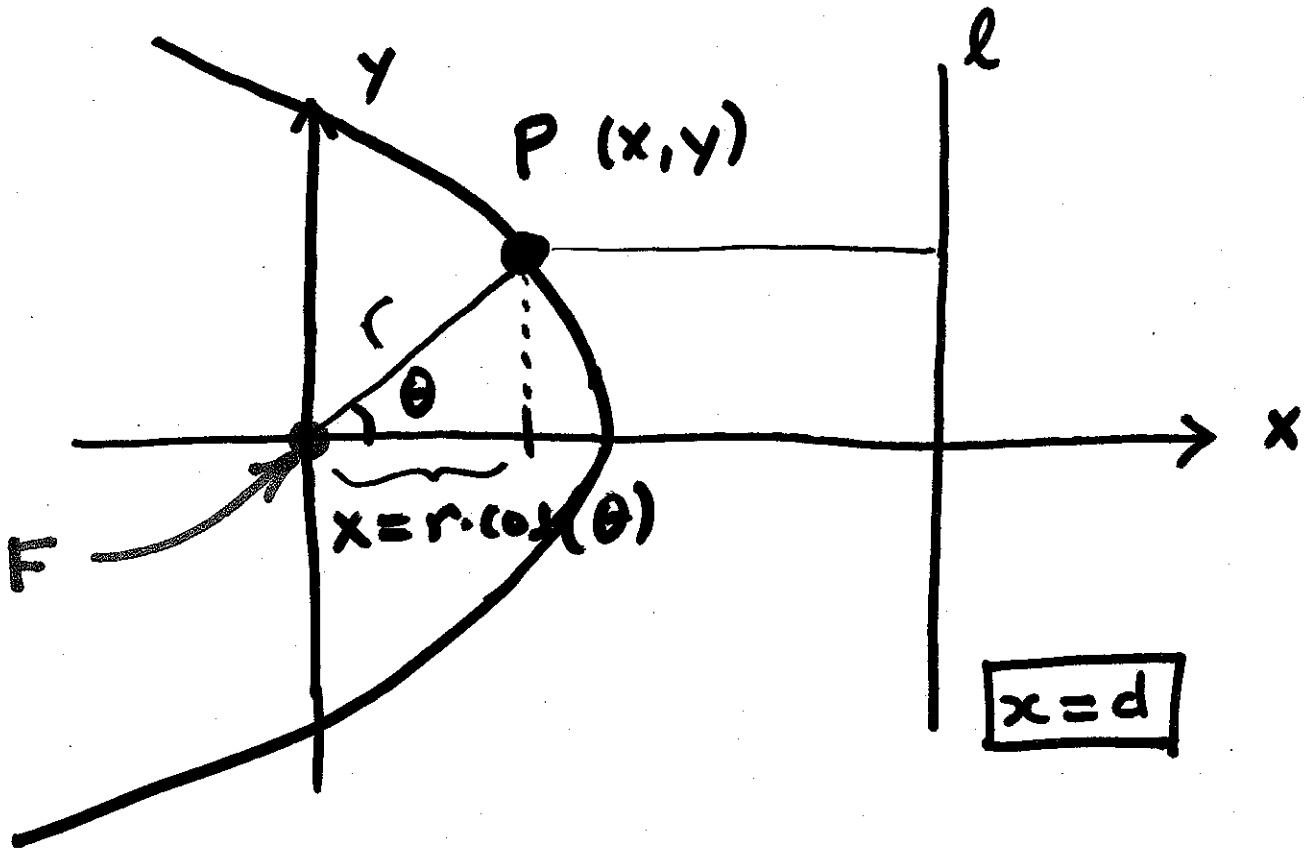
Theorem: Let F be a fixed point and ℓ a fixed line in the plane. The curve that consists of the set of all points, P, satisfying:

$$\frac{\text{Distance from } P \text{ to } F}{\text{Distance from } P \text{ to } \ell} = \frac{|PF|}{|P\ell|} = e$$

where 'e' is a fixed positive number, is a conic section.

$0 < e < 1$	Ellipse
$e = 1$	Parabola
$e > 1$	Hyperbola

Creating a Polar Equations for Conic Sections.



Theorem: $$\frac{|PF|}{|P\ell|} = e$$

$$\frac{r}{d - r\cdot\cos(\theta)} = e$$

$$r = (e)(d - r \cdot \cos\theta)$$

$$r + r \cdot e \cdot \cos(\theta) = ed$$

$$\boxed{r = \frac{e \cdot d}{1 + e \cdot \cos(\theta)}}$$