Outline

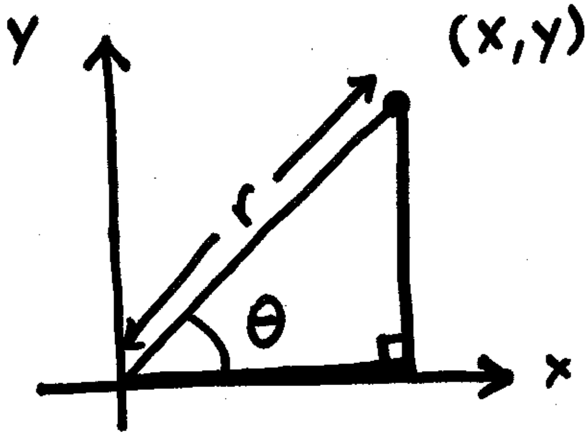
1. Definitions in polar coordinates.

2. Converting between polar and Cartesian equations.

3. Regions in polar coordinates.

4. Tangent lines in polar coordinates.

1. Definitions in Polar Coordinates



$$x = r \cdot \cos(\theta)$$

$$y = r \cdot \sin(\theta)$$

$$r^2 = x^2 + y^2$$

$$\tan(\theta) = \frac{y}{x}$$

- Polar coordinates: r, θ
- Cartesian coordinates: x, y

2. Converting Between Polar and Cartesian Coordinates

(a) Polar to Cartesian

Strategy: ① Express polar equation as a combination of: r^2, $r\cdot\cos(\theta)$, $r\cdot\sin(\theta)$

② Replace: r^2 by x^2+y^2

$r\cdot\cos(\theta)$ by x

$r\cdot\sin(\theta)$ by y

③ Rearrange Cartesian equation so it is easy to use.

Example

Describe each of the following curves in the x-y plane.

(a) $r = \tan(\theta)\cdot\sec(\theta)$

(b) $r = \csc(\theta)$

(c) $r = 6\cdot\sin(\theta)$

(d) $r = \sin(2\theta)$

Solution

(a) $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\qquad \sec(\theta) = \dfrac{1}{\cos(\theta)}$

$$r = \frac{\sin(\theta)}{\cos^2(\theta)}$$

$$r \cdot \cos^2(\theta) = \sin(\theta)$$

$$(r \cdot \cos(\theta))^2 = r \cdot \sin(\theta)$$

$$x^2 = y$$

$r = \tan(\theta) \cdot \sec(\theta)$ is the parabola $y = x^2$.

(b) $r = \csc(\theta)$

$$r = \frac{1}{\sin(\theta)}$$

$$r \cdot \sin(\theta) = 1$$

$$y = 1$$

$r = \csc(\theta)$ is the horizontal line $y = 1$.

(c) $r = 6 \cdot \sin(\theta)$

$$r^2 = 6 \cdot r \cdot \sin(\theta)$$

$$x^2 + y^2 = 6y$$

$$x^2 + y^2 - 6y + 9 = 9$$

$$x^2 + (y-3)^2 = 9.$$

$r = 6 \cdot \sin(\theta)$ is the circle with center $(x, y) = (0, 3)$ and radius 3.

(d) $r = \sin(2\theta)$

$$r = 2 \cdot \sin(\theta) \cdot \cos(\theta)$$

$$r^3 = 2 \cdot r \cdot \sin(\theta) \cdot r \cdot \cos(\theta)$$

$$(x^2+y^2)^{3/2} = 2xy$$

$$(x^2+y^2)^3 = 4x^2\cancel{y}^2$$

(b) <u>Cartesian to Polar</u>

① Replace x by $r \cdot \cos(\theta)$.
Replace y by $r \cdot \sin(\theta)$.

② Rearrange to make r the subject.

<u>Example</u>

Convert the following to polar equations.

(a) $y = 2x + 1$
(b) $x^2 + y^2 = 16x$
(c) $x = y^3$

Solution

(a) $y = 2x + 1$

$$r \cdot \sin(\theta) = 2r\cos(\theta) + 1$$

$$r \cdot (\sin(\theta) - 2\cos(\theta)) = 1$$

$$r = \frac{1}{\sin(\theta) - 2\cos(\theta)}$$

(b) $x^2 + y^2 = 16x$

$$r^2\cos^2(\theta) + r^2\sin^2(\theta) = 16r\cos(\theta)$$

$$r^2(\cos^2(\theta) + \sin^2(\theta)) = 16r\cos(\theta)$$

$$r^2 = 16r\cos(\theta)$$

$$r = 16 \cdot \cos(\theta)$$

(c) $x = y^3$

$$r \cdot \cos(\theta) = r^3 \cdot \sin^3(\theta)$$

$$r^2 = \frac{\cos(\theta)}{\sin^3(\theta)}$$

$$r^2 = \cot(\theta) \cdot \csc^2(\theta)$$

$$r = \pm\sqrt{\cot(\theta) \cdot \csc^2(\theta)}$$

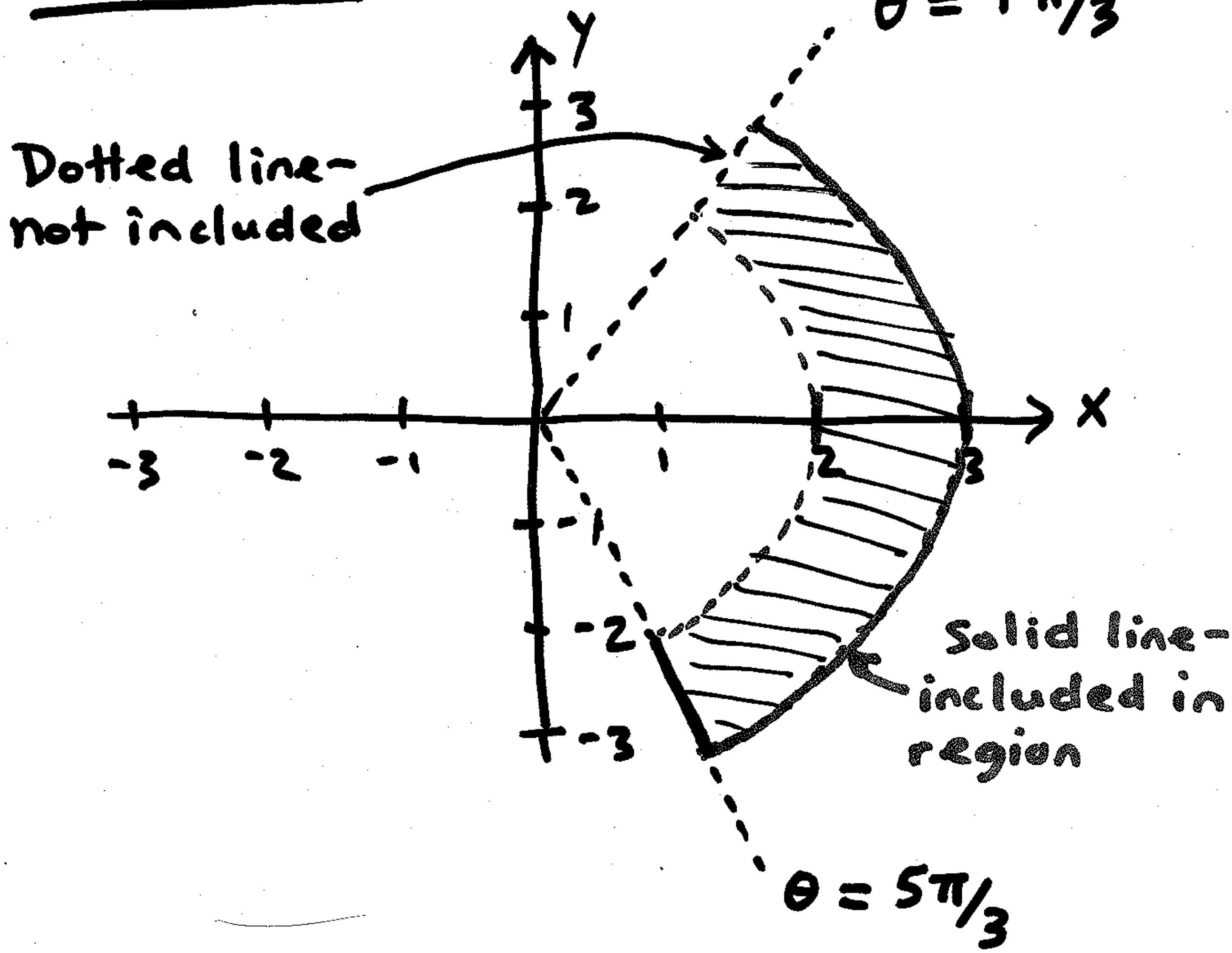
3. Regions Described by Polar Coordinates

Example

Sketch the region in the x-y plane specified by:

$$2 < r \leq 3 \qquad \frac{5\pi}{3} \leq \theta < \frac{7\pi}{3}$$

Solution



y
x
3
2
1
-1
-2
-3
-3
-2
-1
1
2
3
$\theta = 7\pi/3$
$\theta = 5\pi/3$
Dotted line- not included
Solid line- included in region