Outline

1. Spherical coordinates.
2. Triple integrals in spherical coordinates.
3. Recitation on Tuesday.

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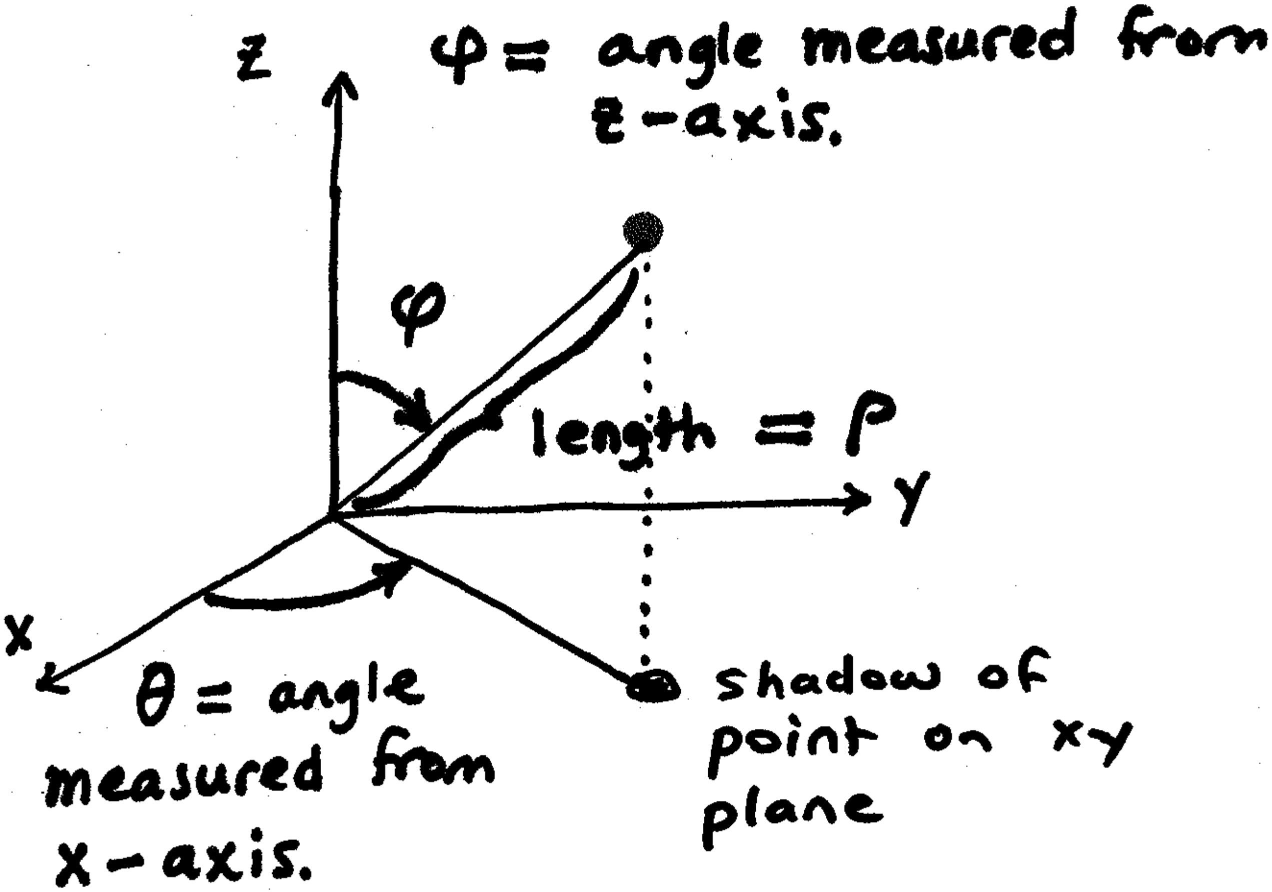
1. Spherical Coordinates

- Use two angles, θ and φ, and distance from the origin ρ.



- Conversion Formulas

$$x = \rho \cdot \cos(\theta) \cdot \sin(\varphi)$$

$$y = \rho \cdot \sin(\theta) \cdot \sin(\varphi)$$

$$z = \rho \cdot \cos(\varphi)$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$dx\,dy\,dz = \rho^2 \cdot \sin(\varphi) \cdot d\rho\, d\theta\, d\varphi$$

2. Triple Integrals in Spherical Coordinates

- Set up the triple integral in spherical coordinates when:

  (a) 3D region is built from spherical shapes.

  (b) When the formula you have to integrate involves $\sqrt{x^2+y^2+z^2}$ or $x^2+y^2+z^2$.

(c) The factor of ρ^2 makes antidifferentiation possible.

e.g. $\iiint_D e^{-(x^2+y^2+z^2)^{3/2}} \, dV \leftarrow$ very difficult.

$$= \iiint_D \underbrace{e^{-\rho^3} \cdot \rho^2}_{\text{now a u-substitution}} \cdot \sin(\varphi) \, d\rho \, d\theta \, d\varphi$$

(d) The factor of $\sin(\varphi)$ makes antidifferentiation possible.

(e) When all else has failed.

<u>Example</u>

Set up a triple integral for the mass of the solid bounded by:

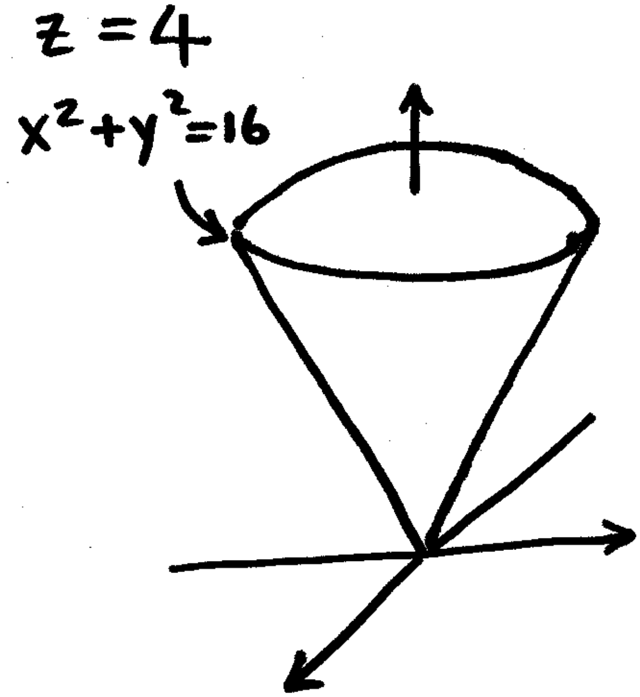
- $x^2 + y^2 + z^2 = 32$
- $z = \sqrt{x^2 + y^2}$

when: (a) $\delta(x, y, z) = z$

(b) $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

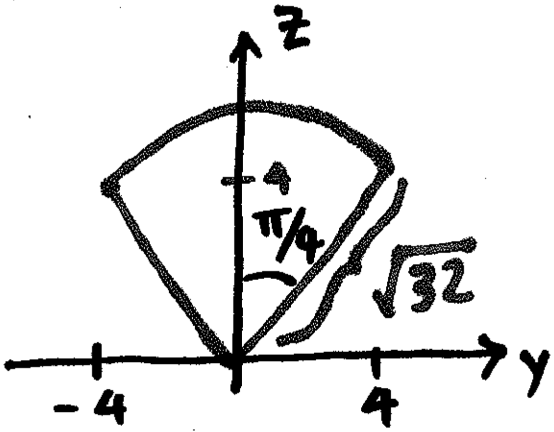
Solution

3D Shape:

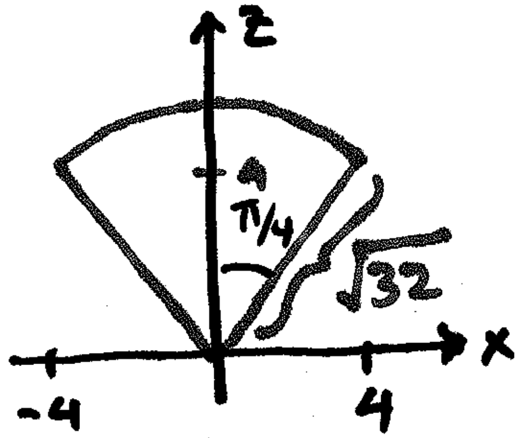


Draw shadows in all coordinate planes:

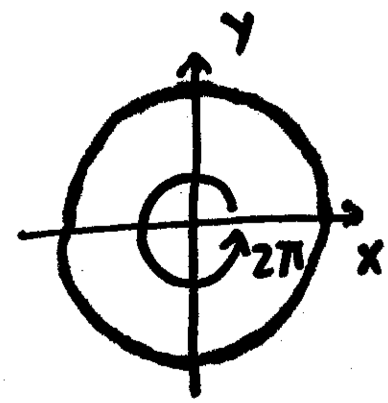
yz:



xz:



xy:



Limits for ρ: $0 \le \rho \le \sqrt{32}$

Limits for φ: $0 \le \varphi \le \frac{\pi}{4}$

or $-\frac{\pi}{4} \le \varphi \le \frac{\pi}{4}$

Limits for θ: $0 \le \theta \le 2\pi$

or $0 \le \theta \le \pi$

(a) $\delta(x, y, z) = z = \rho \cdot \cos(\varphi)$

$$\text{Mass} = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{32}} \rho \cdot \cos(\varphi) \cdot \rho^2 \cdot \sin(\varphi)\, d\rho\, d\varphi\, d\theta$$

(b) $\delta(x, y, z) = \sqrt{x^2+y^2+z^2} = \rho.$

$$\text{Mass} = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{32}} \rho \cdot \rho^2 \cdot \sin(\varphi)\, d\rho\, d\varphi\, d\theta$$