Outline

1. Trochoid.
2. Derivatives and parametric equations.
3. Tangent lines
4. Arc lengths.

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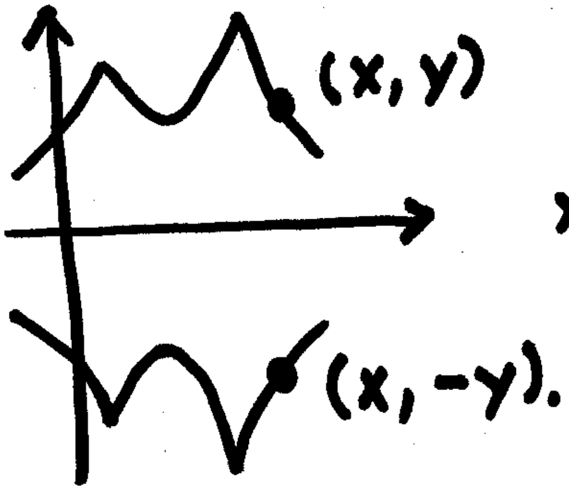
M, W 4-6pm Cyert B6A

Sun-Thurs 8:30pm - 11:00pm in Mudge Library, Donner Reading Room.

Monday 8:30pm - 11:00pm 21-259.

$x(t) = 0.$ y-int.

$y(t) = 0.$ x-int.



x-axis symmetry.

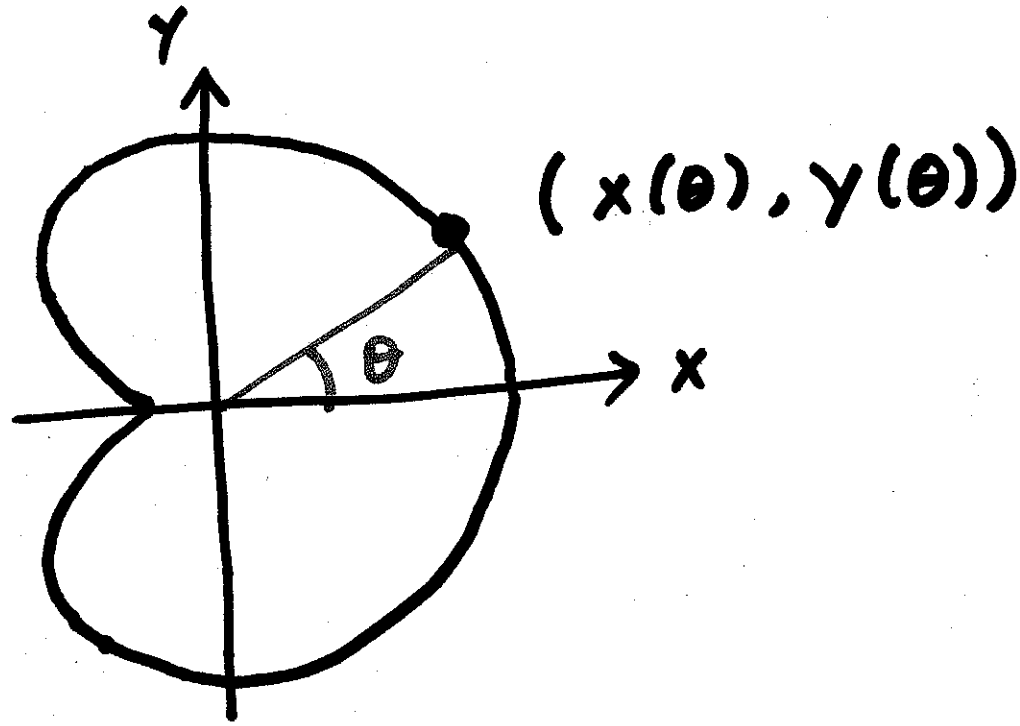
$$(x(t), y(t))$$

$$(x(-t), y(-t)) = (x(t), -y(t)).$$

$$x(t) = x(-t)$$

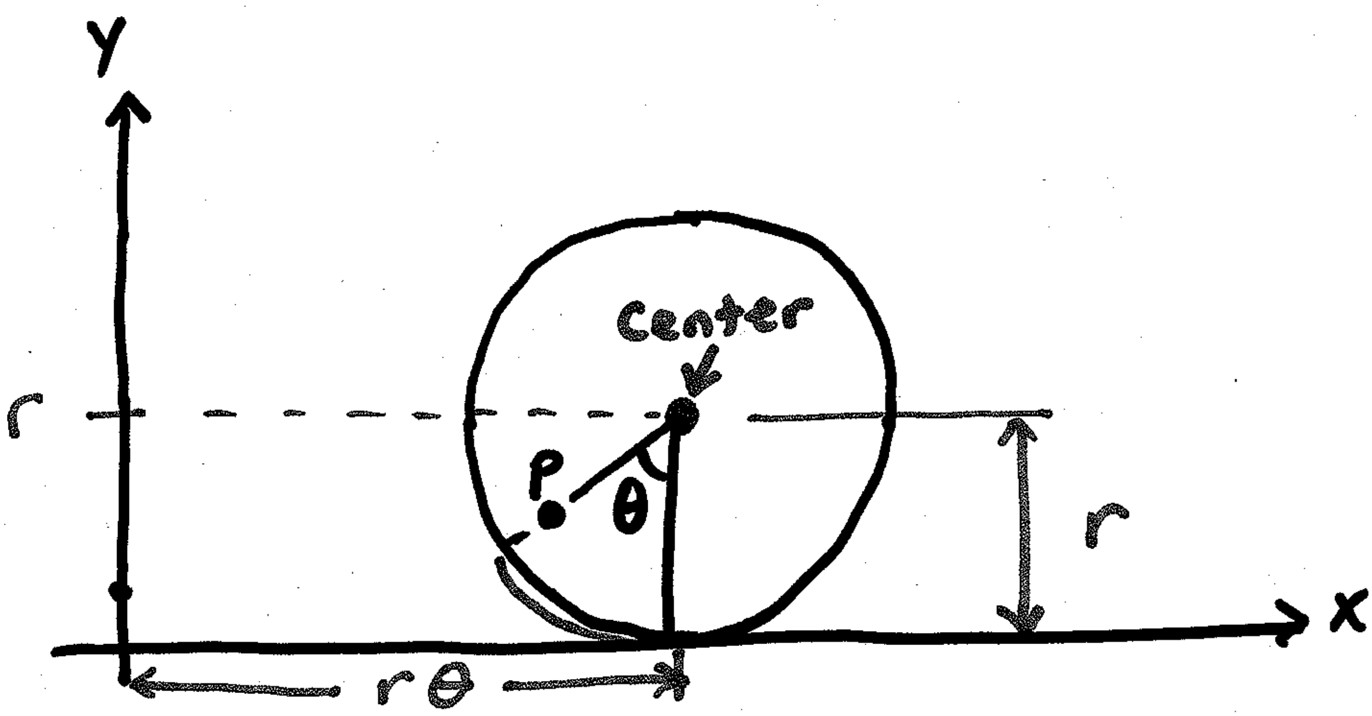
$$-y(t) = y(-t)$$

#53 p. 495.



$$\text{Area} = \frac{1}{2}\int_0^{2\pi}\left(x(\theta)^2 + y(\theta)^2\right) d\theta$$

1. Equation of Trochoid



radius of circle = r
distance from center of circle to P = d.

<u>Goal:</u> Write down equations for

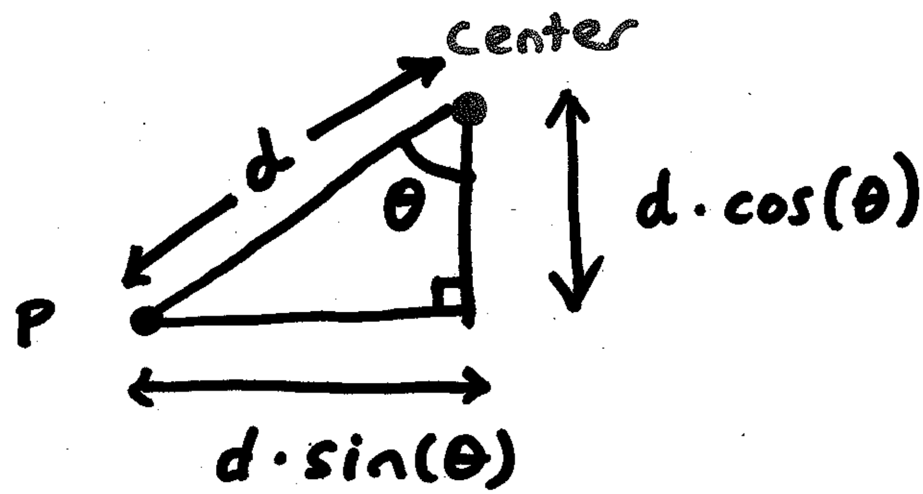
$x(\theta) =$ x co-ord of P.

$y(\theta) =$ y co-ord of P.

Coordinates of center of circle:

$$x = r \cdot \theta$$

$$y = r$$



Coordinates of the point P:

$$x = r \cdot \theta - d \cdot \sin(\theta)$$

$$y = r - d \cdot \cos(\theta)$$

These are the parametric equations of the trochoid.

2. Calculating dy/dx From Parametric Equations

- when we have a curve described by $x(t)$ and $y(t)$:

  - **Speed:** $s(t) = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

  - **Velocity:** $\vec{v}(t) = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}$

    $= \langle x'(t), y'(t) \rangle$

  - **Slope of tangent line in x-y plane:** $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

- Horizontal Tangent line : $\frac{dy}{dt} = 0.$

- Vertical Tangent line : $\frac{dx}{dt} = 0.$

<u>**Example**</u>

$x(t) = 3t^2+1 \qquad y(t) = 2t^3+1$

Find tangent line to point $(4,3)$.

<u>**Solution**</u>

Value of t: $\quad 3t^2+1 = 4$

$\qquad\qquad\qquad 2t^3+1 = 3$

Solution : $\boxed{t=1}$

Calculate dy/dx :

$$\frac{dx}{dt} = 6t \qquad \left.\frac{dx}{dt}\right|_{t=1} = 6.$$

$$\frac{dy}{dt} = 6t^2 \qquad \left.\frac{dy}{dt}\right|_{t=1} = 6.$$

$$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy}{dx}\right|_{(x,y)=(4,3)} = \frac{6}{6} = 1.$$

Final answer:

$$y - 3 = 1 \cdot (x - 4).$$

3. Arc Length

If $x(t)$ and $y(t)$ are the parametric equations giving the position of a particle at time t, the distance covered by the particle between $t=a$ and $t=b$ is:

$$\text{Distance} = \int_a^b s(t)\,dt$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

<u>Example</u>

Find the length of the curve:

$$x(t) = \frac{t}{1+t} \qquad x'(t) = \frac{1\cdot(1+t) - t\cdot(1)}{(1+t)^2}$$

$$y(t) = \ln(1+t)$$

between $t=0$ and $t=2$.

<u>Solution</u>

$$\frac{dx}{dt} = \frac{1}{(1+t)^2} \qquad \frac{dy}{dt} = \frac{1}{1+t}$$

$$\text{Arc length} = \int_0^2 \sqrt{\frac{1}{(1+t)^4} + \frac{1}{(1+t)^2}}\,dt$$

$$u = 1 + t$$

$$\text{Arc length} = \int_1^3 \frac{1}{u^2} \cdot \sqrt{1+u^2}\, du$$

$$= \left[-\frac{\sqrt{1+u^2}}{u} + \ln\left(u + \sqrt{1+u^2}\right) \right]_1^3$$

$$= -\frac{\sqrt{10}}{3} + \ln\left(3+\sqrt{10}\right) + \sqrt{2} - \ln\left(1+\sqrt{2}\right)$$