Outline

1: Setting up triple integrals.

2. Mass.

3. Center of mass.

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HW due tomorrow.

1. Setting Up Triple Integrals

- Have to set up 3 sets of limits of integration.
- Can reduce this to more like a 1+2 dimensional problem rather than a 3D problem (which is much harder).

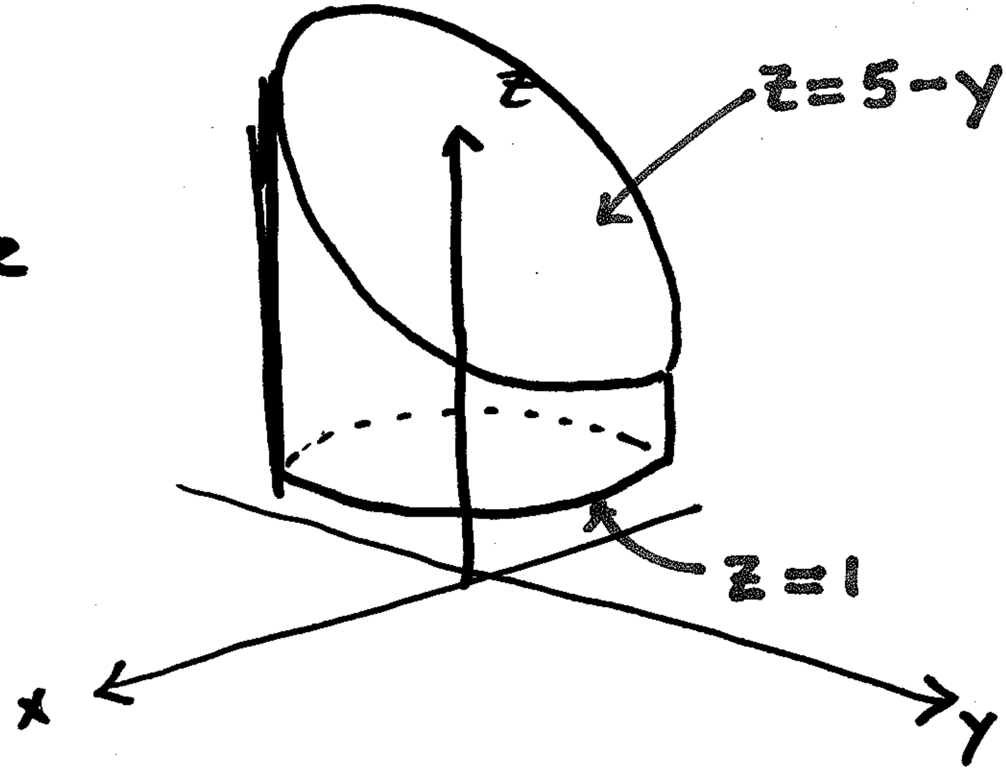
Example

Find the volume enclosed by:

- $z = 1$ Plane.
- $x^2 + y^2 = 9$ Cylinder.
- $z + y = 5$. Plane.

Solution

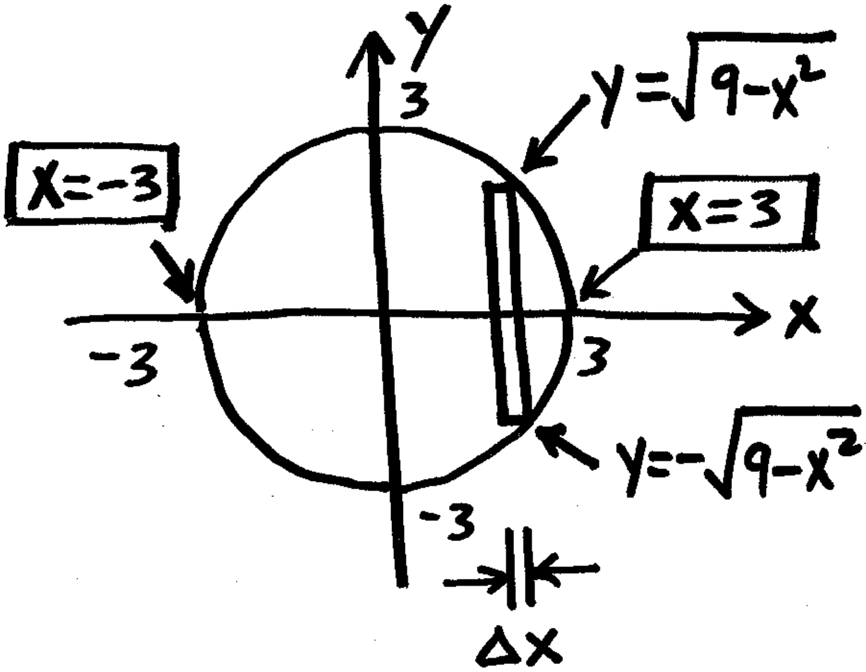
Sketch the volume :



- Selected one of three dimensions, z.
- Values of z at ends of shape are $z=1$ and $z=5-y$.
- These are the limits of integration for dz, <u>but</u> dz has to be the first integral we evaluate (innermost).

Next, consider shadow in the coordinate plane formed by the remaining two dimensions (x and y).

<u>Shadow</u> <u>in</u> xy <u>plane</u>:



- Now just like finding limits for a double integral

always 1 for volume.

$$\text{Volume} = \int_{x=-3}^{x=+3} \int_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}} \int_{z=1}^{z=5-y} 1 \, dz\, dy\, dx$$

$$= 36\pi$$

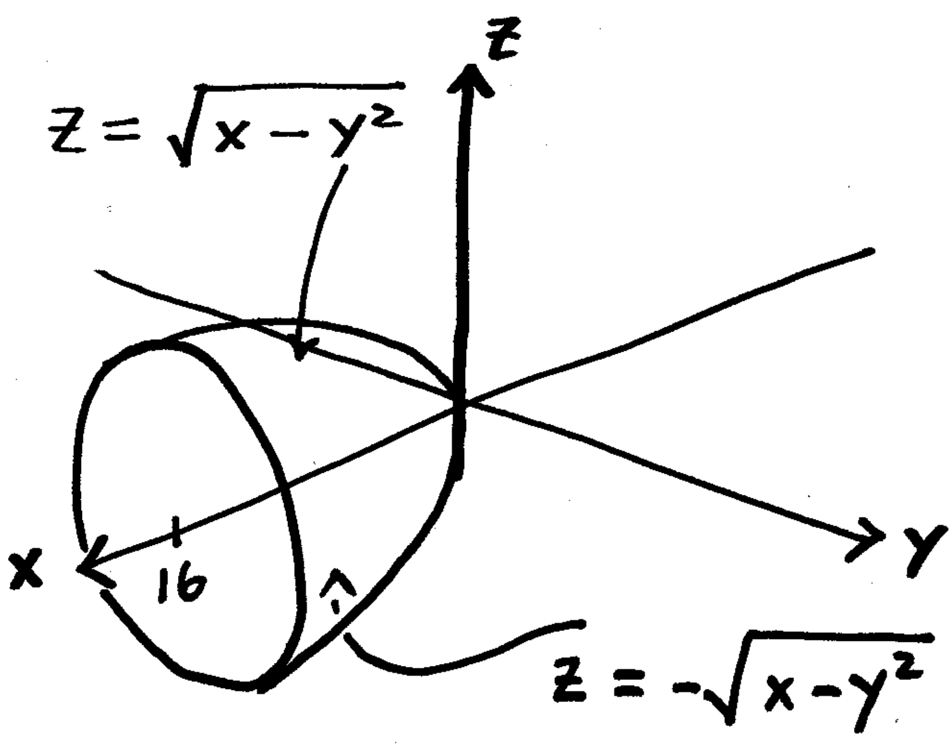
Example

Find the volume bounded by:

- $x = y^2 + z^2$ Paraboloid
- $x = 16.$ Plane.

Solution

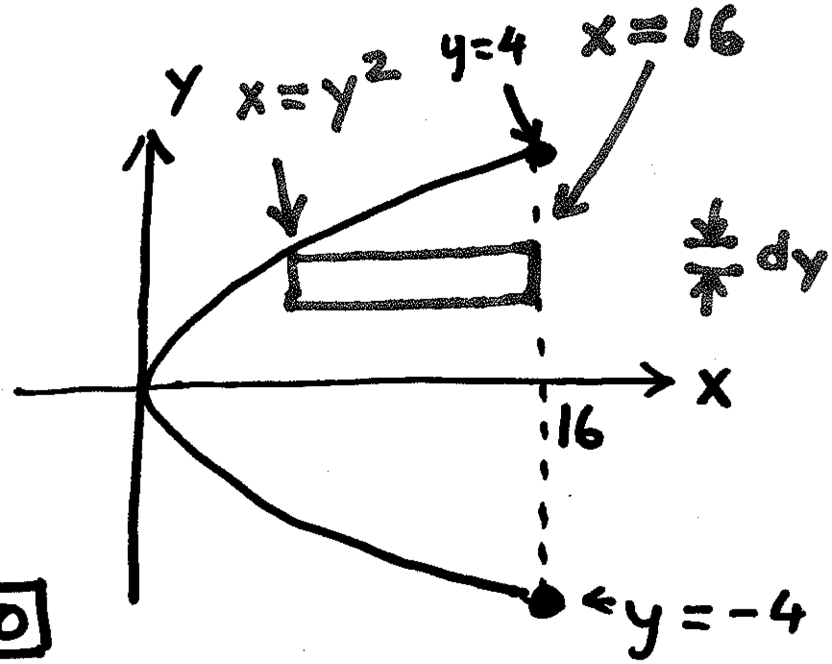
Sketch the volume :



First, isolate z:

- Limits of integration for z are: $\pm\sqrt{x - y^2}$
- dz is the innermost integral.

Next, shadow cast in the xy-plane:



$$x = y^2 + z^2$$

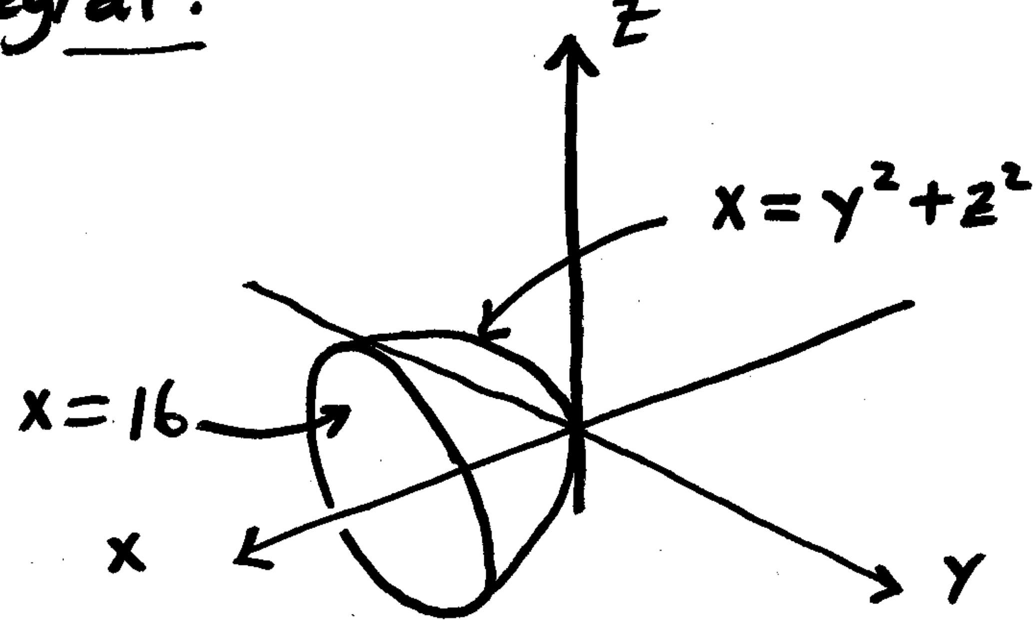
$$\boxed{z=0}$$

$$x = y^2$$

$$\text{Volume} = \int_{y=-4}^{y=+4} \int_{x=y^2}^{x=16} \int_{z=-\sqrt{x-y^2}}^{z=\sqrt{x-y^2}} 1 \cdot dz\,dx\,dy$$

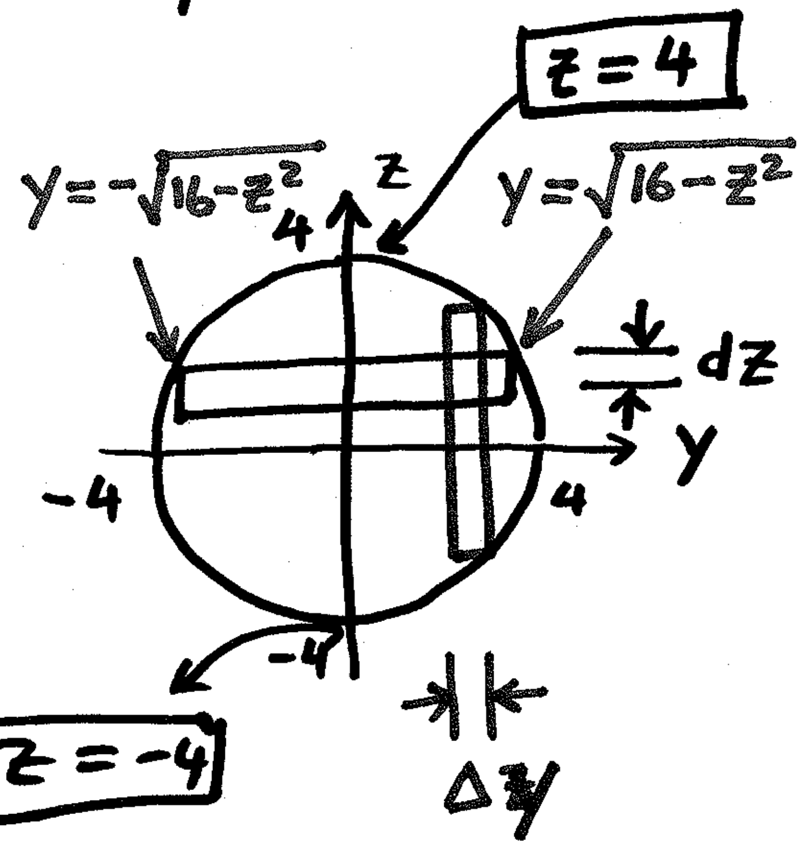
Alternative way to set up this integral:

Sketch of volume:



- dx is the innermost integral.
- Limits of integration are $x = 16$ and $x = y^2 + z^2$.

The shadow on the yz plane is:



$$\text{Volume} = \int_{z=-4}^{z=4} \int_{y=-\sqrt{16-z^2}}^{y=\sqrt{16-z^2}} \int_{x=y^2+z^2}^{x=16} 1 \, dx \, dy \, dz$$

2. Mass

- Density $= \delta(x,y,z)$
- Object occupies a 3D region T.

$$\text{Mass of object} = \iiint_{T} \delta(x,y,z)\, \underbrace{dV}_{\substack{dx\,dy\,dz \text{ in whatever} \\ \text{order you set up}}}$$

the limits of integration for the region, T.

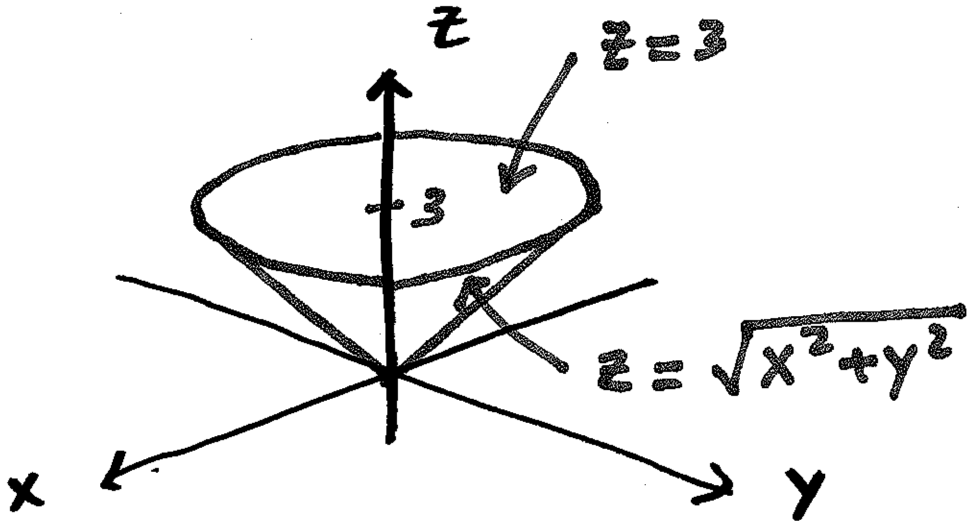
Example

Set up an integral for the mass of the object bounded by:

- $z = \sqrt{x^2 + y^2}$
- $z = 3$

with density $\delta(x,y,z) = z$.

Solution



Shadow in xy plane is a circle of radius 3.

$$\text{Mass} = \int_{x=-3}^{x=3} \int_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}} \int_{z=\sqrt{x^2+y^2}}^{3} z \, dz\, dy\, dx$$

$$= \frac{81\pi}{4}$$