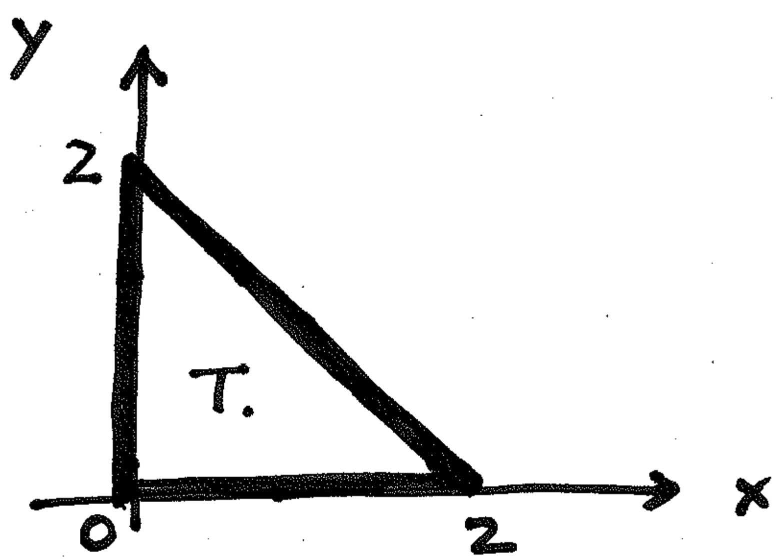
Searching the Boundary

Find global max, min for:

$$f(x,y) = x^2 + y^2 - 2x - y$$

on:



So far: $\frac{\partial f}{\partial x} = 0$ & $\frac{\partial f}{\partial y} = 0$ at $(1, 1/2)$.

Partial derivatives always defined.

Left to do: Search the boundary for points where either:

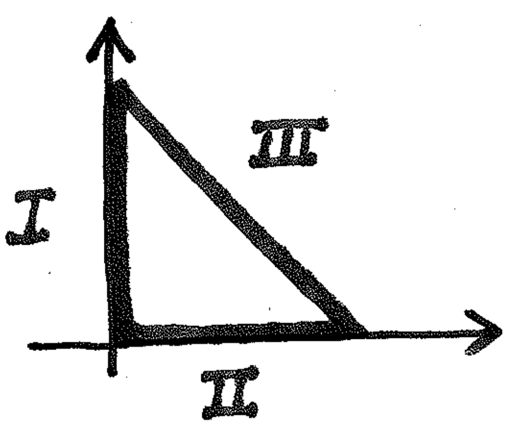
(a) Ordinary derivative = 0

(b) Sharp corner.

For the region T, the sharp corners are:

$$(0,0) \quad , \quad (2,0) \quad , \quad (0,2).$$

<u>Points where ordinary derivative is equal to zero.</u>



I: $x = 0$

II: $y = 0$

III: $y = 2 - x$

- On edge I:

$$f(\underset{0}{x},y) = y^2 - y = f_I(y).$$

$$f_I'(y) = 2y - 1 = 0$$

at the point $(0, \frac{1}{2})$.

- On edge II:

$$f(x,y) = x^2 - 2x = f_{II}(x).$$

$$f_{II}'(x) = 2x - 2 = 0$$

at the point $(1, 0)$.

- On edge III:

$$f(x,y) = x^2 + (2-x)^2 - 2x - (2-x)$$

$$= 2x^2 - 5x + 2$$

$$= f_{III}(x).$$

$$f_{III}'(x) = 4x - 5 = 0$$

at the point $(5/4, 3/4)$

$y = 2-x$.

• To find global max and global min of:

$$f(x,y) = x^2 + y^2 - 2x - y$$

on T:

x	y	f(x,y)	Comments
1	1/2	-1.25	Global min. $\partial f/\partial x = \partial f/\partial y = 0$
0	1/2	-0.25	$f_{I}'(y) = 0$
1	0	-1	$f_{II}'(x) = 0$
5/4	3/4	-1.125	$f_{III}'(x) = 0$
0	0	0	Corners
0	2	2	Global max corner
2	0	0	Corner.