Outline

1. Concept of parametric equations.
2. Graphing parametric curves.
3. Recognizing familiar curves.
4. Formulas for parametric curves.

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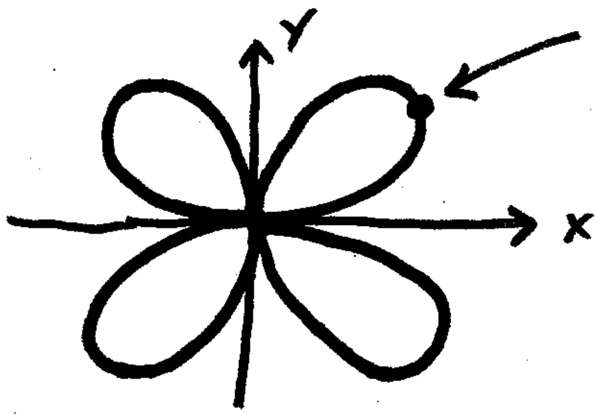
- Quiz in recitation Thursday.

1. Concept of Parametric Equations

- Describing curves in the xy-plane that are not the graphs of functions.



point on curve, use two functions to specify its location.

t = independent variable.

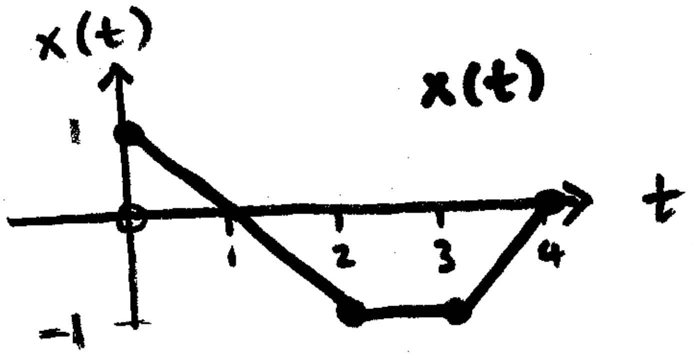
$x(t)$ = x-coordinate of point on curve reached at time t

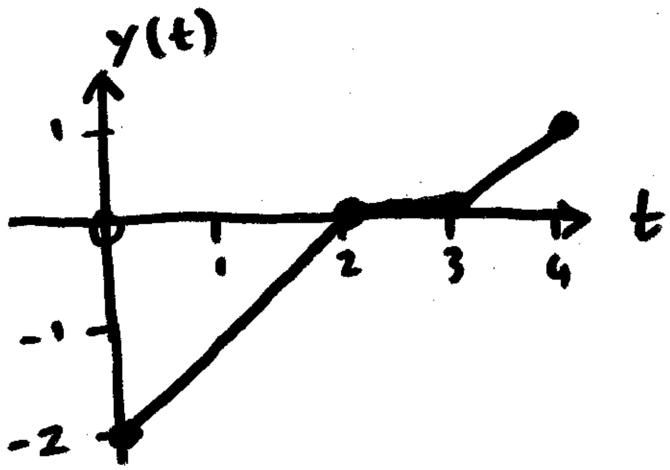
$y(t)$ = y-coordinate of point on curve reached at time t.

2. Graphing a Parametric Curve

Example

Sketch the graph/curve in the x-y plane specified by:

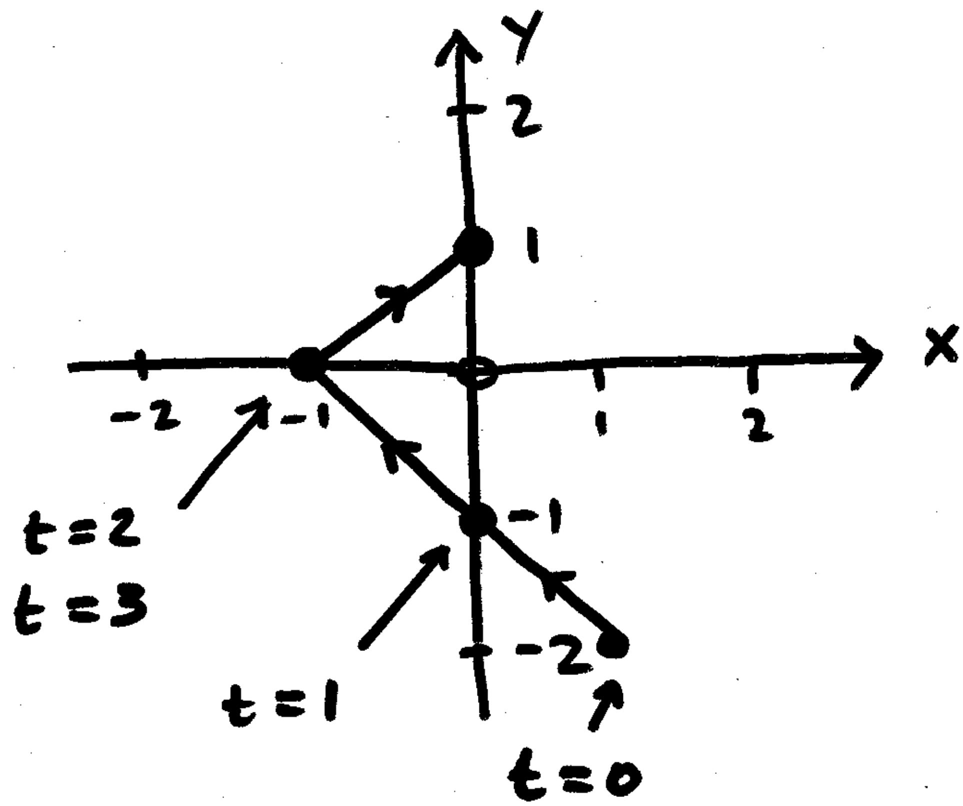




Combine these to give curve in xy-plane $0 \leq t \leq 4$.

Solution

t	0	1	2	3	4
x	1	0	-1	-1	0
y	-2	-1	0	0	1



3. Recognizing Familiar Curves

- Find a relationship between the $x(t)$ and $y(t)$ formulas that removes t from the equation.

Example

What curve do:

$$x(t) = 8 \cdot \cos(t) + 2$$

$$y(t) = 3 \cdot \sin(t) - 1.$$

specify?

Solution

Solve: $x = 8 \cdot \cos(t) + 2$ for t.

$$\frac{x-2}{8} = \cos(t)$$

$$t = \cos^{-1}\left(\frac{x-2}{8}\right).$$

Now plug this into $y(t)$ equation.

$$y = 3 \cdot \sin\left(\cos^{-1}\left(\frac{x-2}{8}\right)\right) - 1.$$

Not a very revealing equation.

Different strategy: find a handy identity that connects the functions within $x(t)$ and $y(t)$.

$$x(t) = 8 \cdot \cos(t) + 2$$

$$\frac{x-2}{8} = \cos(t)$$

$$y(t) = 3 \cdot \sin(t) - 1$$

$$\frac{y+1}{3} = \sin(t)$$

So:

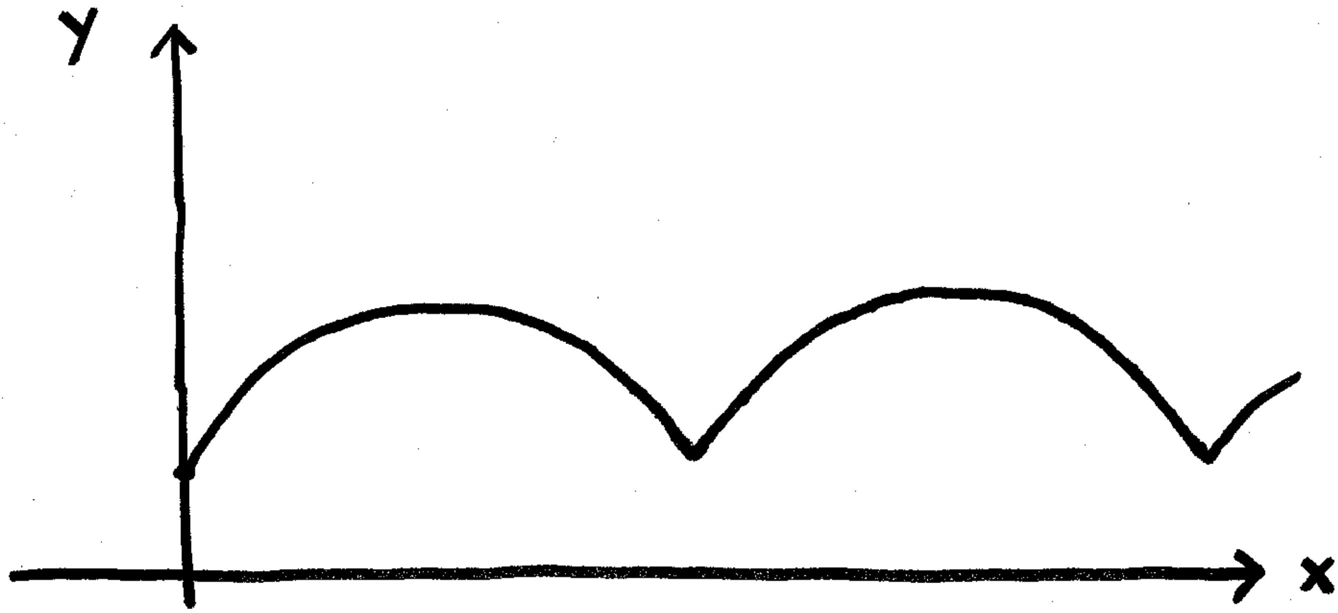
$$\boxed{\left(\frac{x-2}{8}\right)^2 + \left(\frac{y+1}{3}\right)^2 = \boxed{\cos^2(t) + \sin^2(t)} = \boxed{1}}$$

Ellipse: Center at $(2,-1)$

Semi-major axis 8

Semi-minor axis 3.

4. Finding Parametric Equations



This curve is a trochoid.

This is the curve formed when a circle of radius r rolls along the x-axis and we follow the path of a point situated a distance 'd' from the center of the circle.