Outline

1. Contour plots and 3D surfaces.
2. Cylinders.
3. Planes
4. Quadric surfaces.

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- HW #4 due Tuesday
- Do-over test:

Tuesday 2/24/09

8 – 9 pm
9 – 10 pm
} 2210 DH.

1. Contour Plots

- Use graphs in x-y plane to help visualize a 3D graph with a formula like:

$$z = f(x, y).$$

- Contours join points that have the same z-value.

Example

Sketch the graph of:

$z = 15 - x^2 - y^2 + 2x \qquad z \geq 0.$

Solution

Plot curves in xy plane (contours) for fixed values of z.

$z=0$ $\quad 0 = 15 - x^2 - y^2 + 2x$

$x^2 - 2x + 1 + y^2 = 15 + 1$

$(x-1)^2 + y^2 = 16$

$z=1$ $\quad 1 = 15 - x^2 - y^2 + 2x$

$x^2 + 2x + 1 + y^2 = 15$

$(x-1)^2 + y^2 = 15$

$z=20$ $\quad 20 = 15 - x^2 - y^2 + 2x$

$x^2 - 2x + 1 + y^2 = -5 + 1$

$(x-1)^2 + y^2 = -4$

not a contour when x & y are real.

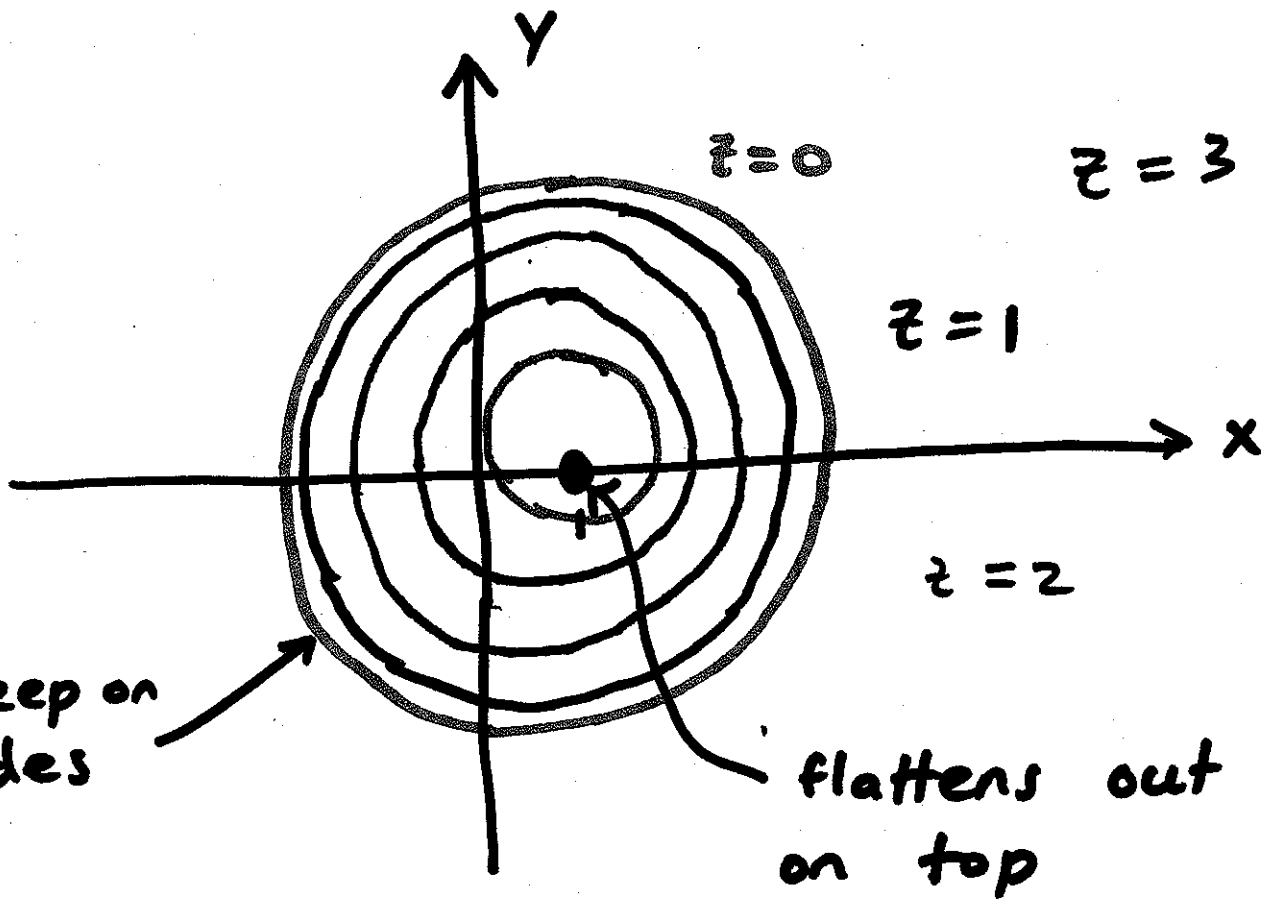
$z = 16$ $\quad 16 = 15 - x^2 - y^2 + 2x$

$$x^2 - 2x + 1 + y^2 = 0$$

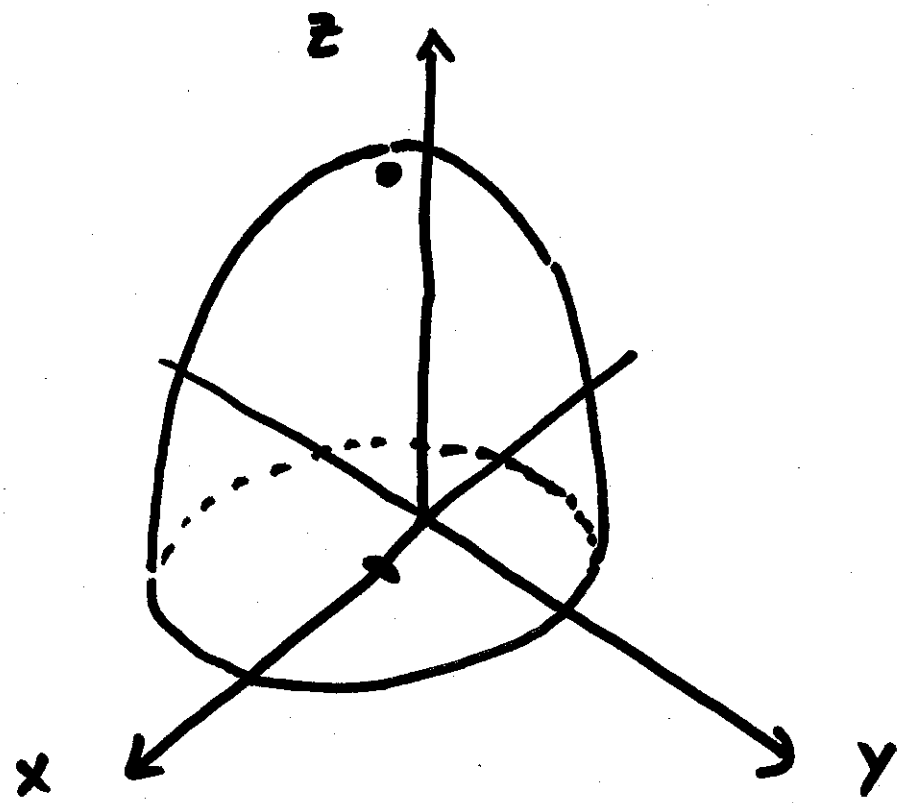
$$(x-1)^2 + y^2 = 0$$

Solution $\quad x = 1 \quad y = 0$ only.

Contour plot :



In 3D !



- Contour plotting is a good tool, but can let you down. If you space the contours too far apart, you may miss some details of graph.
- Reading the algebra of a graph's equation helps us to anticipate the general shape of the graph and not miss details.

2. Cylinders

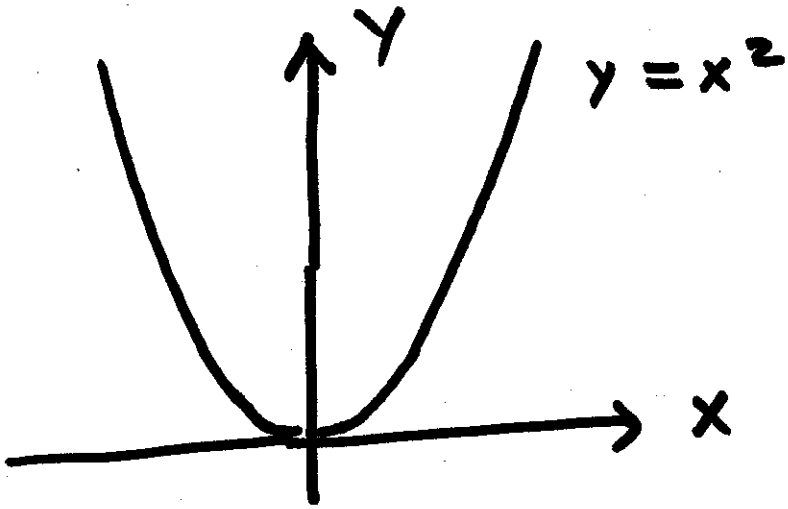
Defn: An equation for a 3D graph in which one of the variables (x, y, z) is not mentioned.

- **Convention:** When variable not mentioned, it can take on any real value.
- Variable not mentioned is the axis that the cylinder extends along.

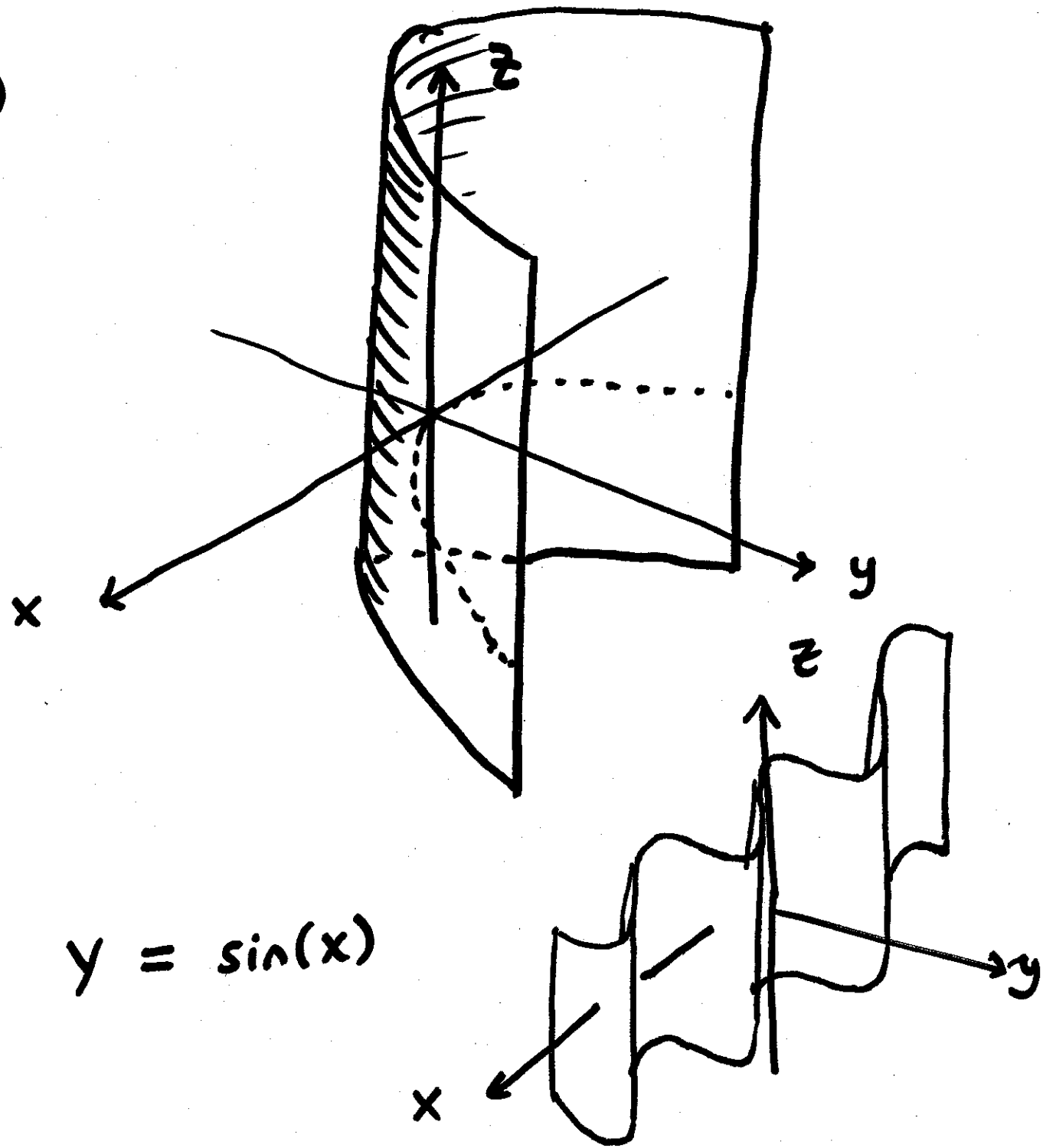
Example

Graph: $y = x^2$ in (a) 2D (b) 3D.

(a)



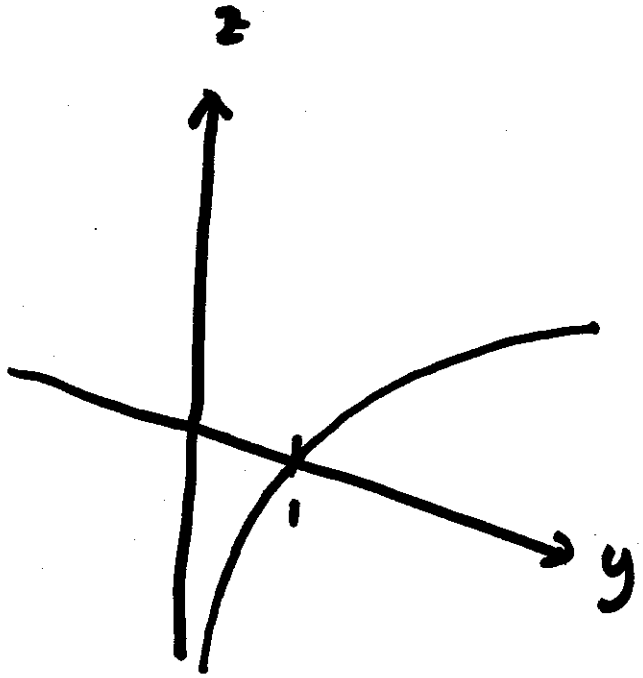
(b)

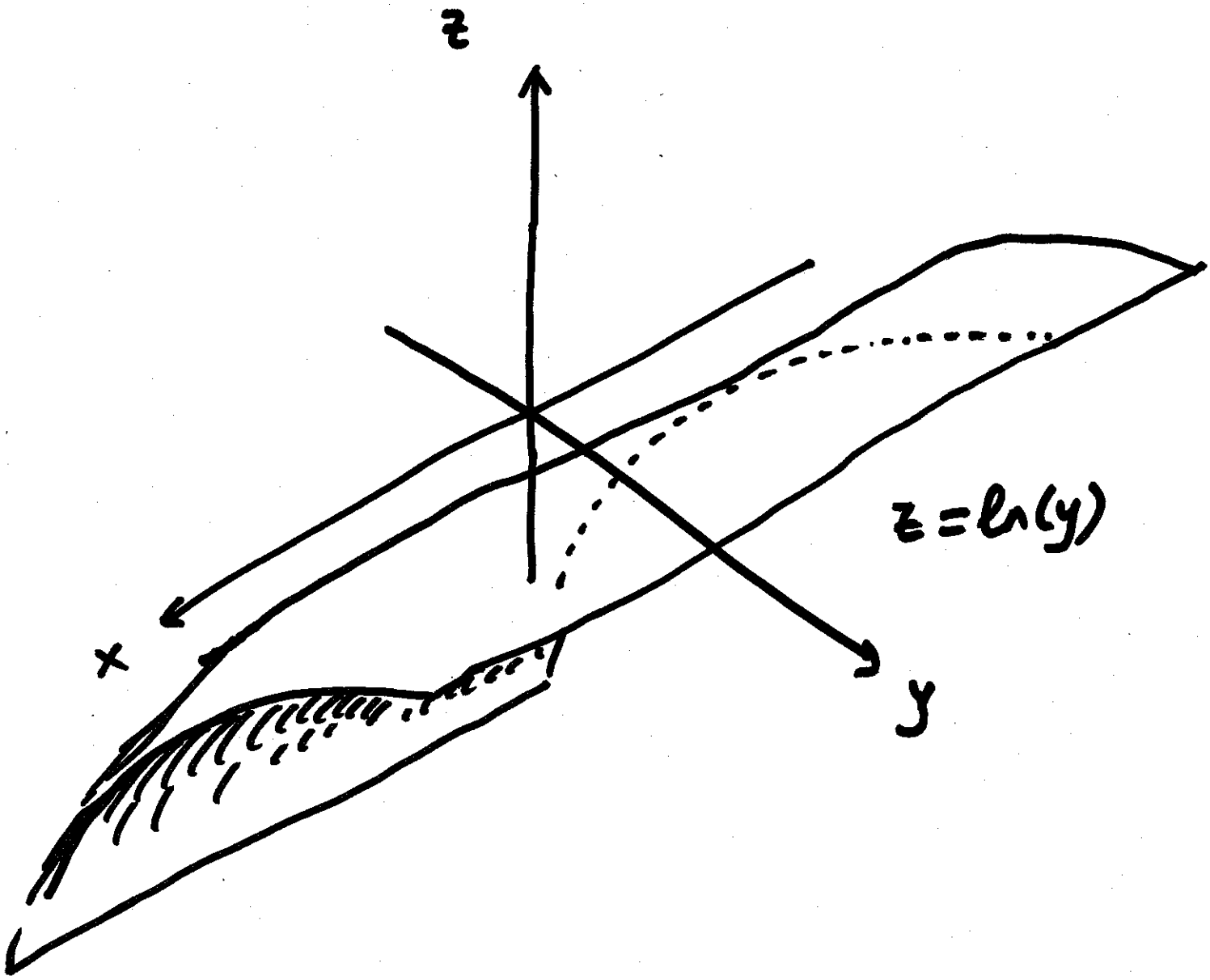


$y = \sin(x)$

Example

$z = \ln(y)$





3. Planes

- Equation of a plane in 3D:

$$a(x-x_0) + b(y-y_0) + c(z-z_0)=0$$

$$ax + by + cz = d$$

where: $d = ax_0 + by_0 + cz_0$

- Sketching a plane.

① Find the intersections of the plane with the:

xy
xz planes.
yz

(These are equations of lines.)

② Draw these lines on a set of 3D axes to form a triangular shape. (This is part of the plane.)

Example

Sketch a graph in 3D of:

$$2(x-1) + 3(y-2) + 4(z-5) = 0$$

$$2x + 3y + 4z = 28$$

Solution

- Intersection with xy-plane ($z=0$)

  Plug $z=0$ into equation for plane we are graphing.

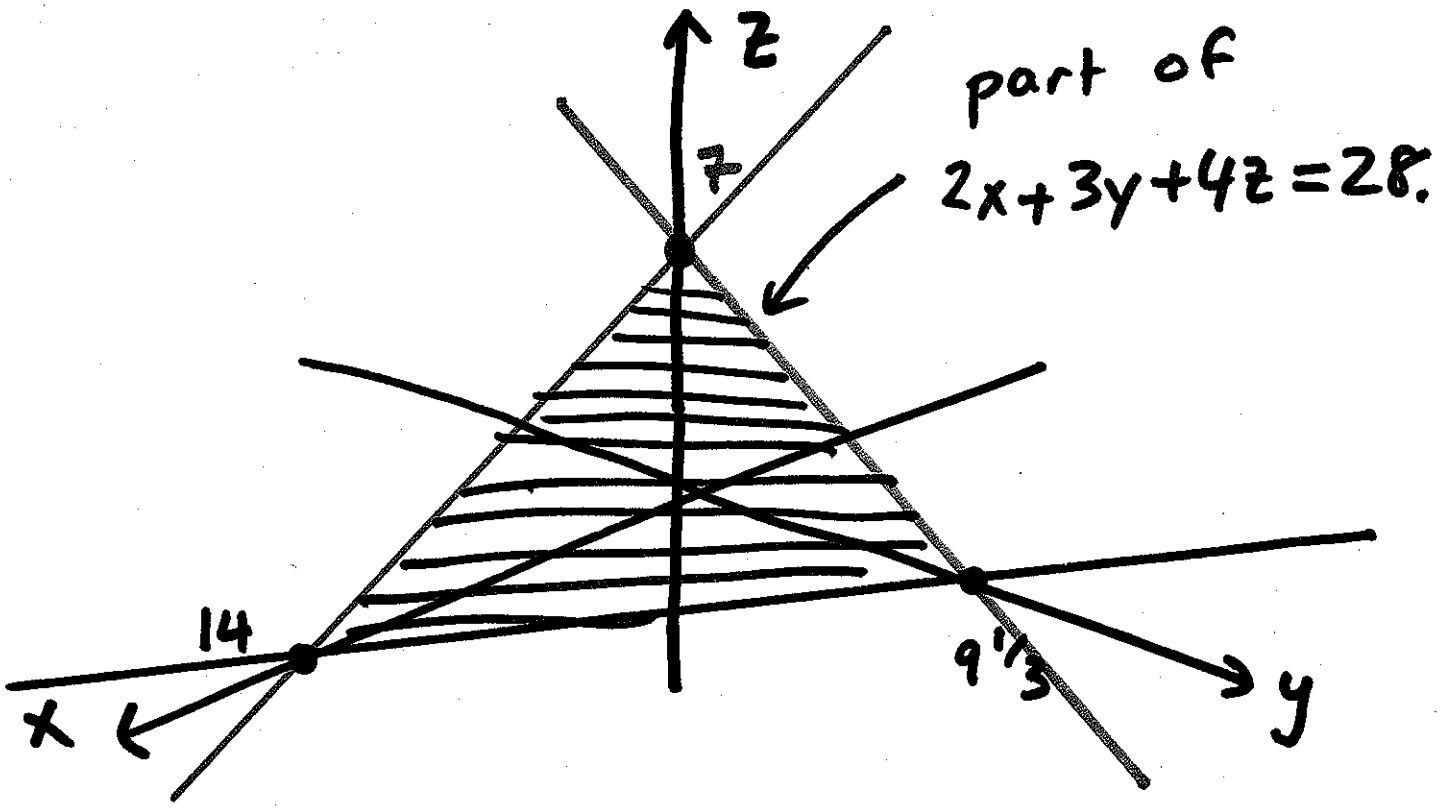
$$\boxed{2x + 3y = 28}$$

- Intersection with xz plane ($y=0$)

$$\boxed{2x + 4z = 28}$$

- Intersection with yz plane ($x=0$)

$$\boxed{3y + 4z = 28}$$


z
7
part of
2x+3y+4z=28.
14
9 1/3
x
y