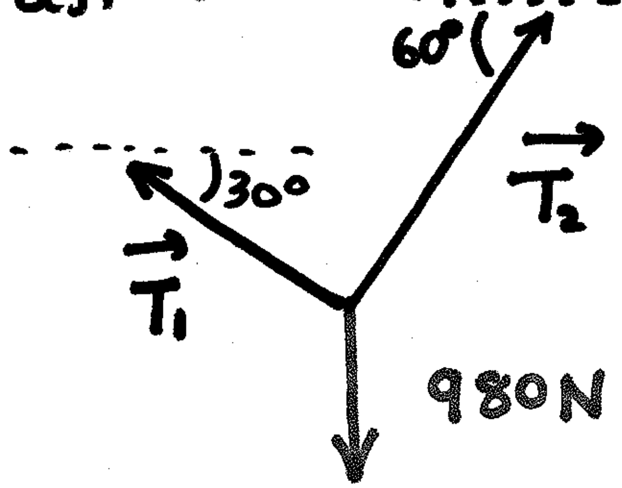
Outline

1. Finish application.
2. Dot product.
3. Angles.
4. Vector projections.

1. Applying Vectors to Physics

From last time........



Sign:

$\uparrow +$

$\rightarrow +$

Horizontal components:

$$-|\vec{T_1}| \cdot \cos(30°)\,\vec{i} + |\vec{T_2}| \cdot \cos(60°)\,\vec{i} = 0\,\vec{i}$$

Vertical components:

$$|\vec{T_1}| \cdot \sin(30°) + |\vec{T_2}| \cdot \sin(60°) - 980 = 0$$

Goal: Find $|\vec{T_1}|$ and $|\vec{T_2}|$.

$$-\frac{\sqrt{3}}{2} \cdot |\vec{T_1}| + \frac{1}{2}|\vec{T_2}| = 0$$

$$\frac{1}{2}|\vec{T_1}| + \frac{\sqrt{3}}{2}|\vec{T_2}| = 980$$

$$|\vec{T}_2| = \sqrt{3} \cdot |\vec{T}_1|$$

$$\tfrac{1}{2}|\vec{T}_1| + \tfrac{3}{2}|\vec{T}_1| = 980$$

$$2|\vec{T}_1| = 980$$

$$|\vec{T}_1| = \frac{980}{2} \text{ N}$$

$$|\vec{T}_2| = \frac{\sqrt{3} \cdot 980}{2}$$

$$= 848.7 \text{ N.}$$

Since the magnitude of each force is less than the 900 N breaking strain of the string, the structure doesn't break.

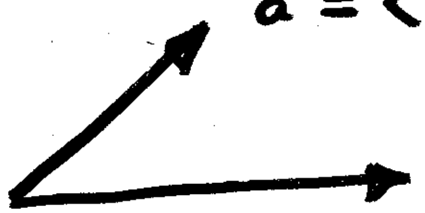
1. Dot Product of Vectors

Def$^{\underline{n}}$: If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and
$\vec{b} = \langle b_1, b_2, b_3 \rangle$ then:

$$\vec{a} \bullet \vec{b} = \underbrace{a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3}_{\text{a real number.}}$$

two vectors

Example



$\vec{a} = \langle 1, 1 \rangle$

$\vec{b} = \langle 2, 0 \rangle$

$$\vec{a} \bullet \vec{b} = \langle 1, 1 \rangle \bullet \langle 2, 0 \rangle = (1)(2) + (1)(0) = 2$$

2. Dot Product, Lengths and Angles in the Plane (and beyond)

Length of a Vector

Prop: Let $\vec{v}$ be a three-dimensional vector. Then:

$$|\vec{v}|^2 \stackrel{?}{=} \vec{v} \bullet \vec{v}$$

Proof: Let $\vec{v} = \langle v_1, v_2, v_3 \rangle$.

$$\text{LHS} = |\vec{v}|^2$$

left hand side of equation

$$= \left\{ \left(\sqrt{v_1^2 + v_2^2 + v_3^2} \right) \right\}^2$$

$$= |v_1^2 + v_2^2 + v_3^2|$$

$$= v_1^2 + v_2^2 + v_3^2$$

$$\text{RHS} = \vec{v} \bullet \vec{v}$$

↑
right hand side of equation

$$= \langle v_1, v_2, v_3 \rangle \bullet \langle v_1, v_2, v_3 \rangle$$

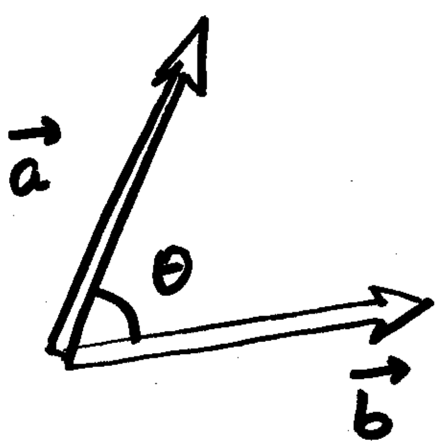
$$= v_1 \cdot v_1 + v_2 \cdot v_2 + v_3 \cdot v_3$$

$$= v_1^2 + v_2^2 + v_3^2$$

Since LHS = RHS the proposition is proven.

<u>**Note:**</u> $|\vec{v}| = \sqrt{\vec{v} \bullet \vec{v}}$

<u>Angle Between Vectors</u>



$$\boxed{\cos(\theta) = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}| \cdot |\vec{b}|}}$$

Neither $\vec{a}$ nor $\vec{b}$ are $\vec{0}$.

Orthogonal Vectors

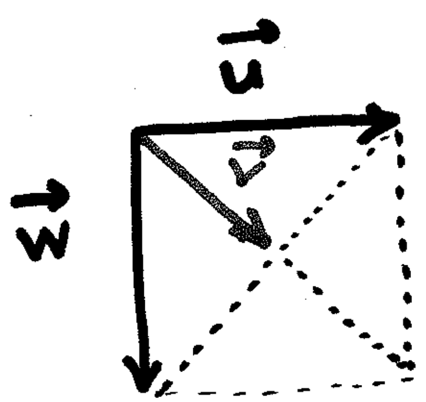
Two vectors $\vec{a}$ and $\vec{b}$ are said to be orthogonal iff:

$$\vec{a} \bullet \vec{b} = 0.$$

Geometrically, this means that the two vectors meet at a right angle ($\pi/2$ or $90°$).

Example

$\vec{u}$ is a unit vector (length = 1).



(This is a square)

What are: (a) $\vec{u} \bullet \vec{w}$ and (b) $\vec{u} \bullet \vec{v}$?

<u>Solution</u>: (a) $\vec{u}$ and $\vec{w}$ are perpendicular, so:

$$\vec{u} \bullet \vec{w} = 0.$$

(b) $|\vec{u}| \cdot |\vec{v}| \cdot \cos(\theta) = \vec{u} \bullet \vec{v}$

(Angle formula for 2 vectors).

$|\vec{u}| = 1 \qquad |\vec{v}| = \sqrt{2}/2$

$\cos(\theta) = \cos(\pi/4) = 1/\sqrt{2}$

$\vec{u} \bullet \vec{v} = (1)\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}.$

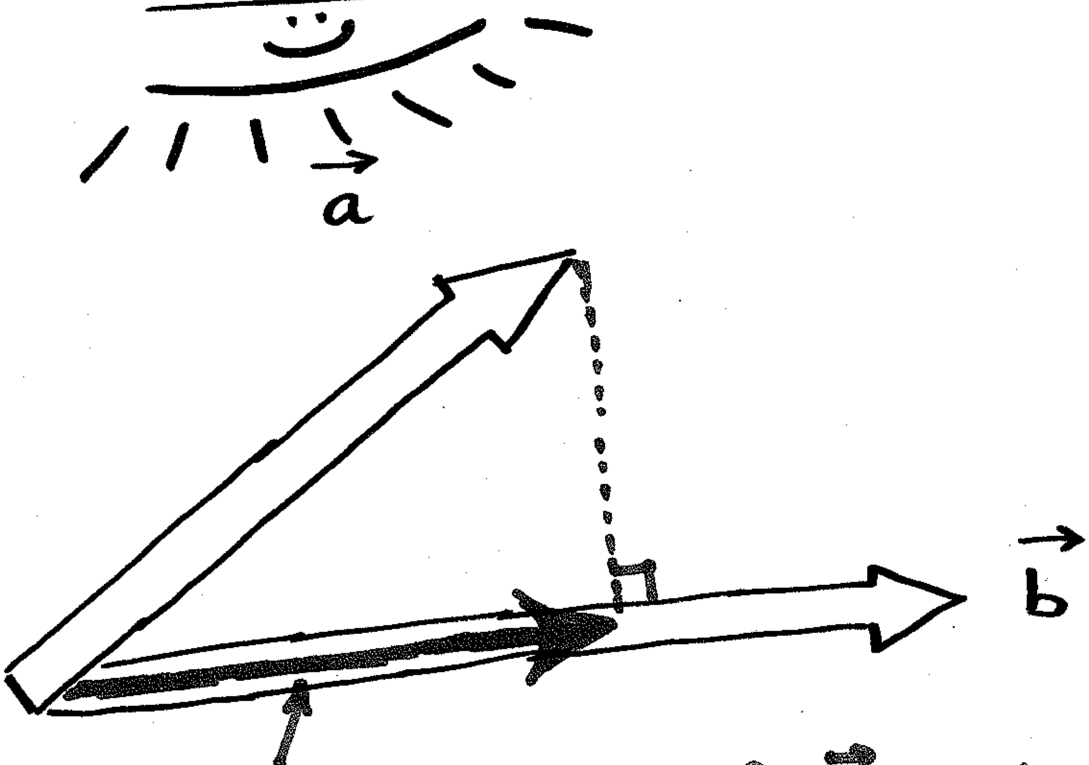
<u>Check</u>: $\vec{v} = \frac{1}{2}(\vec{u} + \vec{w})$

$$\vec{u} \bullet \vec{v} = \frac{1}{2}\left(\underbrace{\vec{u} \bullet \vec{u}} + \underbrace{\vec{u} \bullet \vec{w}}\right)$$

$$= \frac{1}{2}\left(1^2 + 0\right)$$

$$= \frac{1}{2}$$

3. Vector Projection



The projection of $\vec{a}$ onto $\vec{b}$

$$\text{proj}_{\vec{b}}(\vec{a}) = \text{projection of } \vec{a} \text{ onto } \vec{b}$$

$$= \underbrace{\underbrace{\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}}_{\text{real number}} \cdot \underbrace{\vec{b}}_{\text{vector}}}_{\text{vector overall}}$$

this defines a vector that is parallel to $\vec{b}$.