Outline

1. Polar coordinates.

2. Curves in polar coordinates

3. Tangent lines.

Do-over: Thursday 7, 8, 9 pm
2315 DH.

Final: Friday 12/12 8:30 am
Rooms on web site.

Arc Length Example

Find the length of the curve:

$$x(t) = t - \cos(t)$$
$$y(t) = e^t$$

$(0 \le t \le 2\pi)$ between the points $(-1, 1)$ and $(\pi/2, e^{\pi/2})$.

Solution

$$\text{Arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Find t-values that correspond to the points $(-1,1)$ and $(\pi/2, e^{\pi/2})$.

For $(-1,1)$:
$$x(t) = t - \cos(t) = -1$$
$$y(t) = e^t = 1$$

So: $\boxed{a = 0}$

For $(\pi/2, e^{\pi/2})$: $x(t) = t - \cos(t) = \pi/2$

$y(t) = e^t = e^{\pi/2}$

So: $\boxed{b = \pi/2}$.

Next find dx/dt and dy/dt.

$x(t) = t - \cos(t)$ $\qquad \dfrac{dx}{dt} = 1 + \sin(t)$

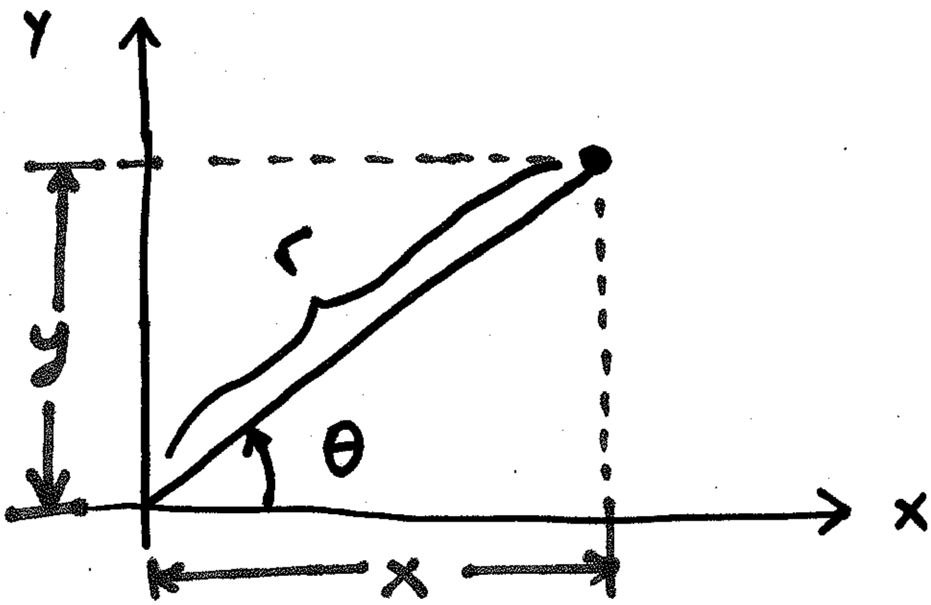
$y(t) = e^t$ $\qquad \dfrac{dy}{dt} = e^t$

$$\text{Arc length} = \int_0^{\pi/2} \sqrt{(1+\sin(t))^2 + (e^t)^2}\, dt$$

$$= 4.63148015829$$

1. Polar Coordinates

- Instead of specifying points in the plane using a horizontal distance (x) and a vertical distance (y), we can give a **bearing** (θ) and a **range** (r).



$x = r \cdot \cos(\theta)$ $\qquad$ $r = \sqrt{x^2 + y^2}$

$y = r \cdot \sin(\theta)$ $\qquad$ $\tan(\theta) = \dfrac{y}{x}$

2. Graphing Curves Expressed in Polar Coordinates

- Curves (lines, etc.) expressed in polar coordinates are usually written as:

$$r = f(\theta) \quad \underline{\text{or}} \quad \theta = g(r)$$

Examples

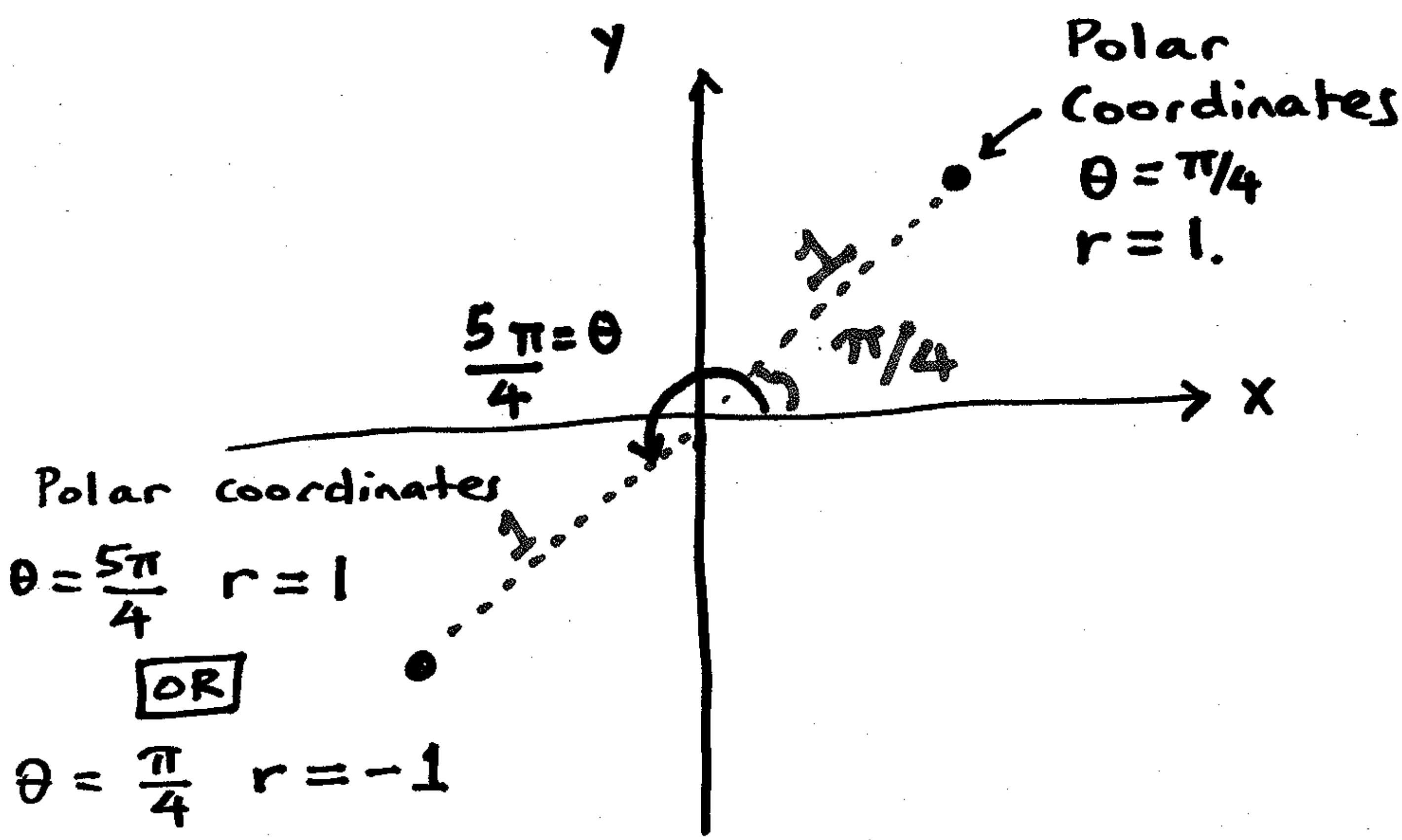
① $\boxed{r = 4}$ Circle of radius 4 with center at $(0,0)$.

Note: Since θ is not explicitly mentioned in this formula, θ is assumed to have any and all real number values.

② $\boxed{\theta = \pi/4}$

Diagonal line that makes a 45° angle with x-axis and passes through (0, 0). (i.e. $y = x$).

<u>Note</u>: Since r is not mentioned, it can be any real number, including a <u>negative</u> number.



- A negative value of r (e.g. $r = -5$) means move that distance (e.g. 5 units out from the origin) but in a direction that is diametrically opposed to the given bearing (i.e. $\theta + \pi$, or $\theta + 180°$).

- A negative value of θ means measure the bearing in a clockwise direction from the x-axis.

Example

Sketch the curve defined by:

$$r = 2 \cdot \sin(\theta)$$

in the x-y plane.

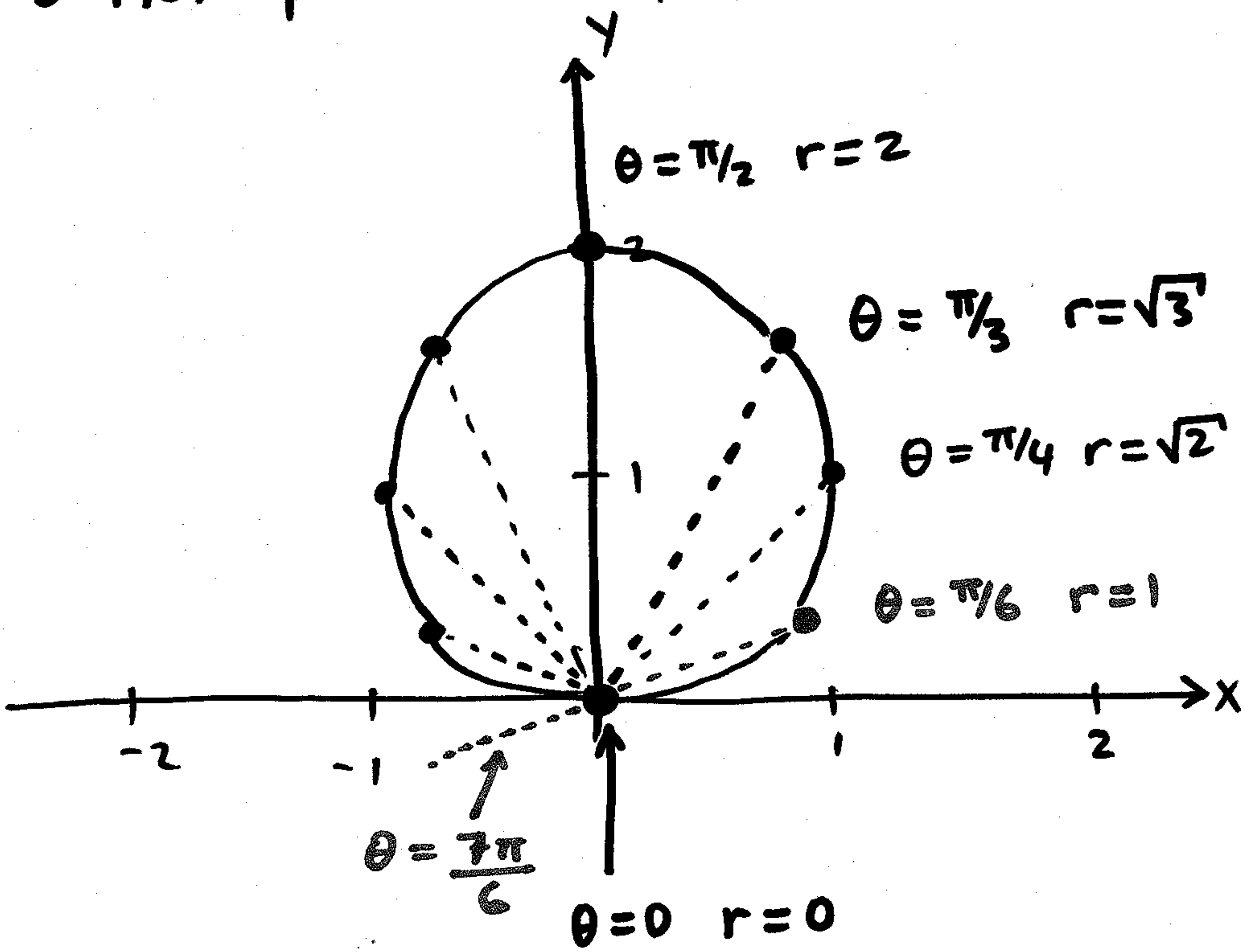
Solution

- First, build tables of r and θ.

θ	r
0	0
$\pi/6$	1
$\pi/4$	$\sqrt{2}$
$\pi/3$	$\sqrt{3}$
$\pi/2$	2

θ	r
$2\pi/3$	$\sqrt{3}$
$3\pi/4$	$\sqrt{2}$
$5\pi/6$	1
π	0
$7\pi/6$	-1

- Plot points on xy plane.



- Join points with a smooth curve.