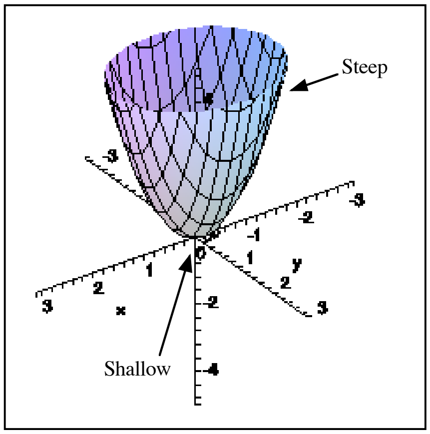
Solutions to Homework #7

Problems from Pages 599-601 (Section 11.1)

22. The contour plot on the left corresponds to the paraboloid and the contour plot on the right corresponds to the cone.

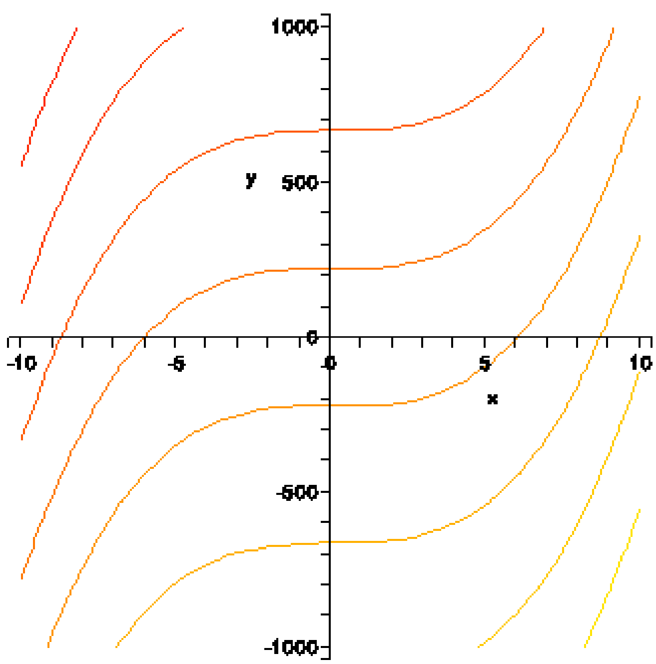


You can tell because the bottom (or top) of a paraboloid has a shallow slope, whereas the sides have steeper slopes.

Shallow slopes correspond to wide spaces between contour lines and steep slopes correspond to small spaces between contour lines. This pattern only appears in the contour plot on the left.

26. The contour plot is shown below. The contours are cubic polynomials with equations of the form:

$$y = x^3 - k.$$



Problems from Page 608 (Section 11.2)

8. I suspect that the limit:

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$$

does not exist because the sum of the powers in the term on the top is the same as the power of both terms in the denominator. To show that this is the case, we will make the substitution $y = mx$:

$$\frac{6x^3y}{2x^4+y^4} = \frac{6x^3(mx)}{2x^4+(mx)^4} = \frac{6mx^4}{2x^4+m^4x^4} = \frac{6m}{2+m^4},\ x \neq 0.$$

As this expression depends on m, we will get different z-values as we approach the point (0, 0) along different paths of the form $y = mx$. This means that the limit as $(x, y) \to (0, 0)$ does not exist.

Problems from Pages 614-617 (Section 11.3)

2. At the point $(x, y) = (2, 1)$ we have $z = 10$. At the point $(x, y) = (3, 1)$ we have $z = 14$ and at the point $(x, y) = (2, 2)$ we have $z = 8$. We approximate the partial derivatives using the difference quotients:

$$\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(2,1)} \approx \frac{14-10}{3-2} = 4 \qquad \text{and} \qquad \left.\frac{\partial z}{\partial y}\right|_{(x,y)=(2,1)} \approx \frac{8-10}{2-1} = -2.$$

4. **(a)** $f_x(-1, 2)$ is negative.

(b) $f_y(-1, 2)$ is negative.

(c) $f_{xx}(-1, 2)$ is positive.

(d) $f_{yy}(-1, 2)$ is negative.

44. The set of all four of the partial derivatives of $f(x, y) = \ln(3x + 5y)$ are as listed below.

$$\frac{\partial^2 f}{\partial x^2} = \frac{-9}{(3x+5y)^2} \qquad \frac{\partial^2 f}{\partial y^2} = \frac{-25}{(3x+5y)^2} \qquad \frac{\partial^2 f}{\partial y \partial x} = \frac{-15}{(3x+5y)^2} \qquad \frac{\partial^2 f}{\partial x \partial y} = \frac{-15}{(3x+5y)^2}$$

56. The six derivatives that need to be computed to obtain the desired expression are as follows:

$$u = x^a \cdot y^b \cdot z^c$$

$$\frac{\partial u}{\partial x} = a \cdot x^{a-1} \cdot y^b \cdot z^c$$

$$\frac{\partial^2 u}{\partial x \partial y} = a \cdot b \cdot x^{a-1} \cdot y^{b-1} \cdot z^c$$

$$\frac{\partial^3 u}{\partial x \partial y^2} = a \cdot b \cdot (b-1) \cdot x^{a-1} \cdot y^{b-2} \cdot z^c$$

$$\frac{\partial^4 u}{\partial x \partial y^2 \partial z} = a \cdot b \cdot (b-1) \cdot c \cdot x^{a-1} \cdot y^{b-2} \cdot z^{c-1}$$

$$\frac{\partial^5 u}{\partial x \partial y^2 \partial z^2} = a \cdot b \cdot (b-1) \cdot c \cdot (c-1) \cdot x^{a-1} \cdot y^{b-2} \cdot z^{c-2}$$

$$\frac{\partial^5 u}{\partial x \partial y^2 \partial z^3} = a \cdot b \cdot (b-1) \cdot c \cdot (c-1) \cdot (c-2) \cdot x^{a-1} \cdot y^{b-2} \cdot z^{c-3}$$

Problems from Pages 624-625 (Section 11.4)

6. The partial derivatives of the function $z = e^{x^2 - y^2}$ at the point where x = 1 and y = −1 are:

$$\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(1,-1)} = 2x \cdot e^{x^2-y^2}\Big|_{(x,y)=(1,-1)} = 2 \quad \text{and} \quad \left.\frac{\partial z}{\partial y}\right|_{(x,y)=(1,-1)} = -2y \cdot e^{x^2-y^2}\Big|_{(x,y)=(1,-1)} = 2.$$

The equation of the tangent plane is then:

$$2(x - 1) + 2(y + 1) - (z - 1) = 0.$$

16. The partial derivatives of the function $z = \ln(x - 3y)$ at the point where x = 7 and y = −1 are:

$$\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(7,2)} = \left.\frac{1}{x-3y}\right|_{(x,y)=(7,2)} = 1 \quad \text{and} \quad \left.\frac{\partial z}{\partial y}\right|_{(x,y)=(7,2)} = \left.\frac{-3}{x-3y}\right|_{(x,y)=(7,2)} = -3.$$

The equation of the tangent plane is:

$$(x - 7) - 3(y - 2) - z = 0.$$

To approximate $f(6.9, 2.06)$ we will rearrange the equation for the tangent plane to make z the subject and substitute $x = 6.9$ and $y = 2.06$ into this equation.

$$f(6.9, 2.06) \approx (6.9 - 7) - 3(2.06 - 2) = -0.28.$$

Problems from Pages 631-633 (Section 11.5)

12. The definition of the function:

$$g(r, s) = f(2r - s, s^2 - 4r)$$

shows that $x = 2r - s$ and $y = s^2 - 4r$. Note that when $(r, s) = (1, 2)$, $(x, y) = (0, 0)$. Using these observations and the Chain Rule for partial derivatives:

$$g_r(1.2) = f_x(0,0)\cdot 2 + f_y(0,0)\cdot(-4) = 8 - 32 = -24.$$

$$g_s(1.2) = f_x(0,0)\cdot(-1) + f_y(0,0)\cdot 2\cdot(0) = -4.$$