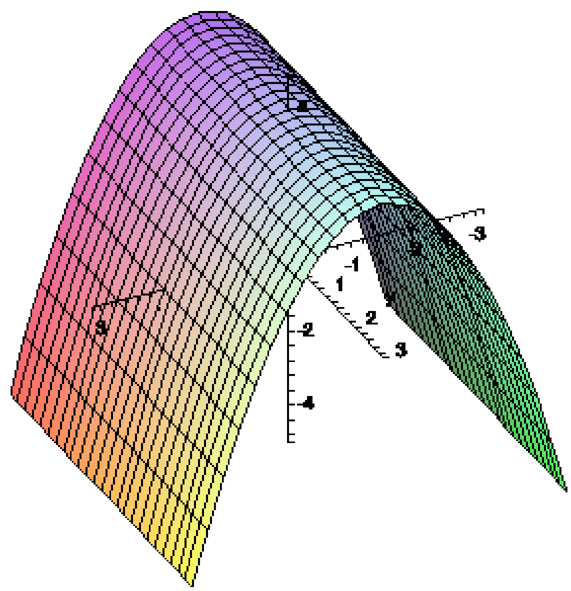
Solutions to Homework #6

Problems from Page 558 (Section 10.6)

4. The surface $z = 4 - x^2$ is a "cylinder" that extends along the y-axis. The cross-section in the xz-plane is a parabola that has a z-intercept of $z = 4$ and opens downwards along the negative z-axis.



10. **(a)** In this problem we will explore the graph of $-x^2 - y^2 + z^2 = 1$. The traces with constant $x = k$ are curves with equations:

$$-y^2 + z^2 = 1 + k^2.$$

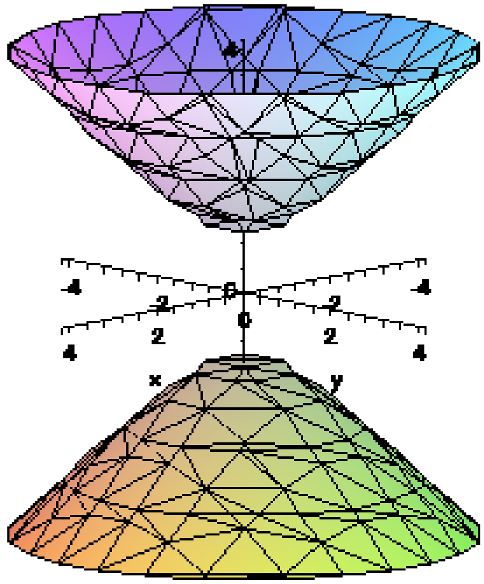
These are hyperbolas. The traces with constant $y = k$ are curves with equations:

$$-x^2 + z^2 = 1 + k^2.$$

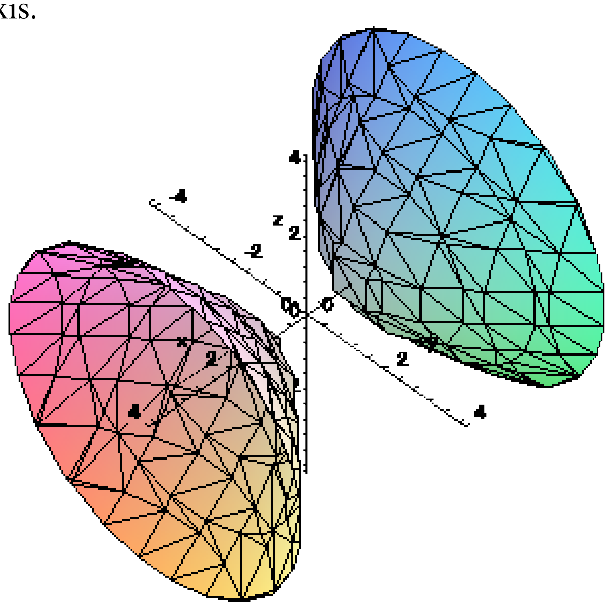
These are also hyperbolas. The traces with constant $z = k$ are curves with equations:

$$x^2 + y^2 = k^2 - 1,$$

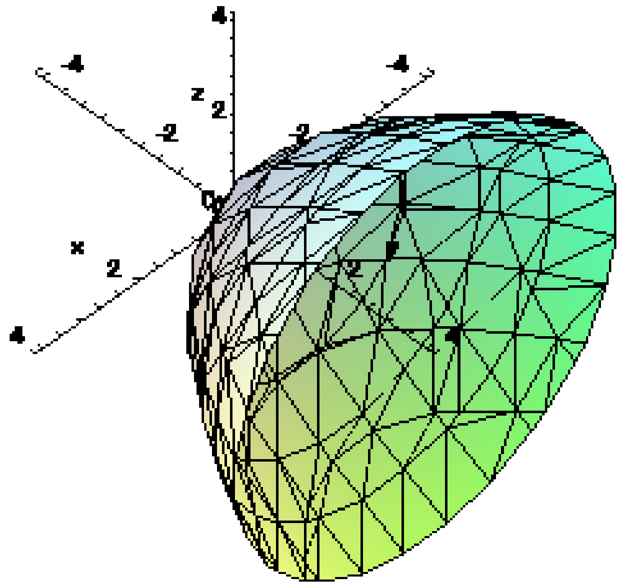
which are circles so long as $|k| > 1$. The traces with $|k| < 1$ are empty. Putting these traces together gives a picture that is a hyperbola of two sheets, as shown below.



(b) When examining the equation of a hyperbola of two sheets, the term that is the "odd one out" gives the axis that the sheets of the hyperbola open along. The graph of $x^2 - y^2 - z^2 = 1$ has the same overall shape as the graph from Part (a) except that the two sheets of the hyperboloid open along the x-axis instead of along the y-axis.



12. The surface $4y = x^2 + z^2$ is a elliptic paraboloid that opens along the positive y-axis. The cross-sections of the graph for constant y are circles.



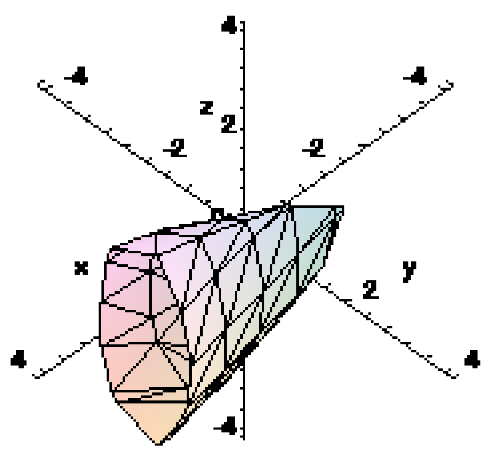
26. We will use the technique of Completing the Square to put the equation:

$$4y^2 + z^2 - x - 16y - 4z + 20 = 0.$$

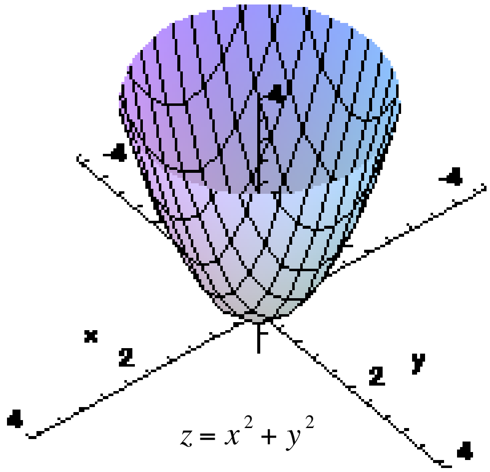
Doing this gives:

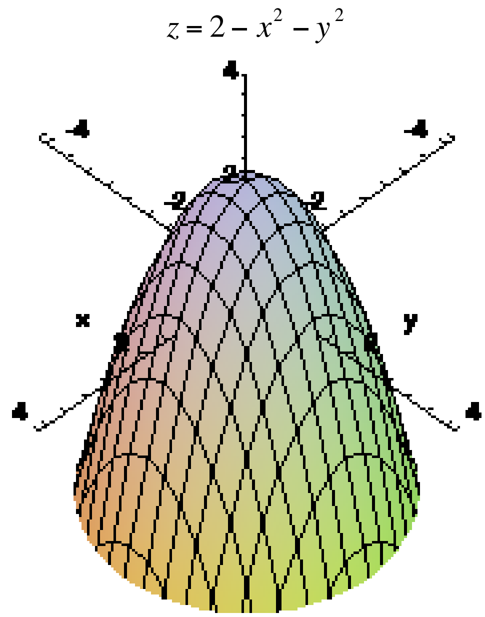
$$\frac{x}{4} = (y-2)^2 + \frac{(z-2)^2}{4}.$$

This curve is an elliptic paraboloid that opens along the positive x-axis. The cross-sections for constant values of x are ellipses that are taller in the z direction than they are wide in the y direction as shown below. The vertex of the paraboloid is located at the point (0, 2, 2).



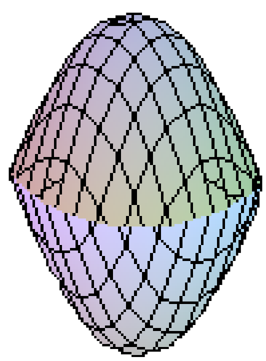
30. The two surfaces are elliptic paraboloids that open along the z-axis, as shown below. The cross-sections of both paraboloids for constant z are circles.





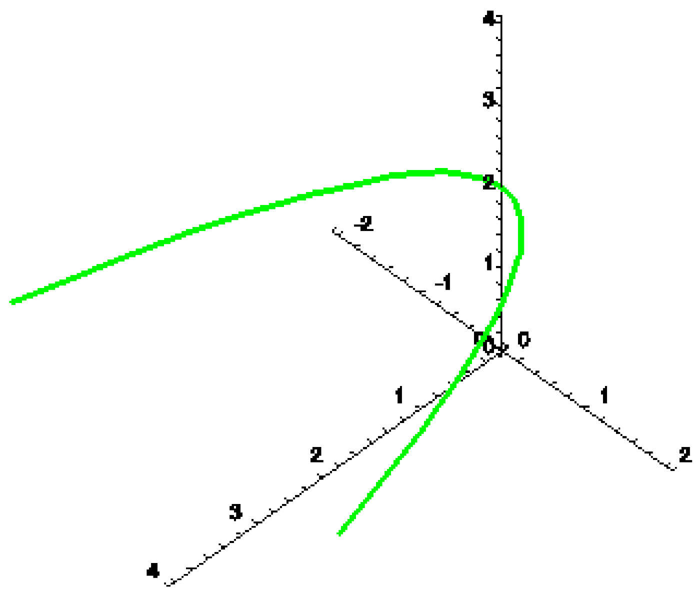
The volume bounded by these two surfaces is illustrated below. The equation of the intersection between the two surfaces is:

$$x^2 + y^2 = 1 \qquad\qquad z = 1.$$



Problems from Pages 568-570 (Section 10.7)

10. The parametric equations are $x = t^2$, $y = t$ and $z = 2$ so the equivalent Cartesian equations are $x = y^2$ and $z = 2$. This curve is a parabola that opens along the x-axis hovering at a height of 2 above the xy-plane, as shown on the next page.



16. First we will find the vector equation. For the position vector we can use either of the two points. We will use <–2, 4, 0>. For the direction vector we can use the vector formed by the difference of the two points: <6, –1, 2>. The vector equation for the line is then:

$$\langle x, y, z\rangle = \langle -2, 4, 0\rangle + t\cdot\langle 6, -1, 2\rangle.$$

The parametric equations are just the components of this vector equation:

$$x = -2 + 6t \qquad y = 4 - t \qquad z = 2t.$$

24. You can simply plug the given parametric equations into the Cartesian equations and verify that they are satisfied.

Parametric equations:

$$x(t) = \sin(t) \qquad y(t) = \cos(t) \qquad z(t) = \sin^2(t)$$

Cartesian equations:

$x^2 + y^2 = 1$: $\quad x(t)^2 + y(t)^2 = \sin^2(t) + \cos^2(t) = 1.$

$z = x^2$: $\quad \sin^2(t) = (\sin(t))^2.$

28. The projection of the curve of intersection onto the *xy*-plane is the circle:

$$x^2 + y^2 = 4.$$

The parametric equations for a circle of radius 2 with center at (0, 0) are:

$$x(t) = 2\cdot\sin(t) \qquad y(t) = 2\cdot\cos(t).$$

The z-coordinate of each point on the curve of intersection is $z = xy$. Substituting the above parametric equations into this Cartesian equation gives the parametric equation for $z(t)$:

$$z(t) = 4\cdot\cos(t)\cdot\sin(t).$$

Problems from Pages 599-601 (Section 11.1)

18. The graph of the function $z = f(x, y) = 3 - x^2 - y^2$ is an elliptic paraboloid that has its z-intercept at the point (0, 0, 3) and opens along the negative z-axis. The cross-sections for constant z are circles. The graph of $z = f(x, y)$ is shown below.