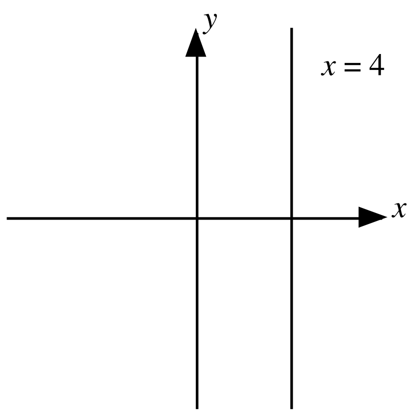
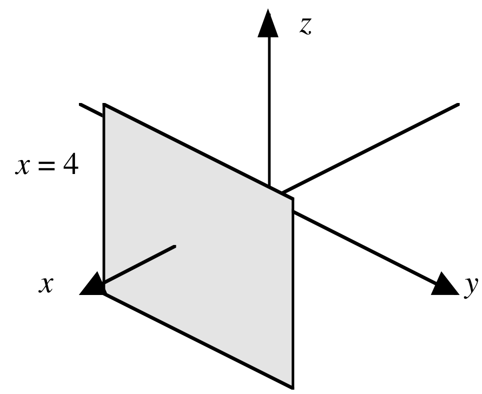
Solutions to Homework #4

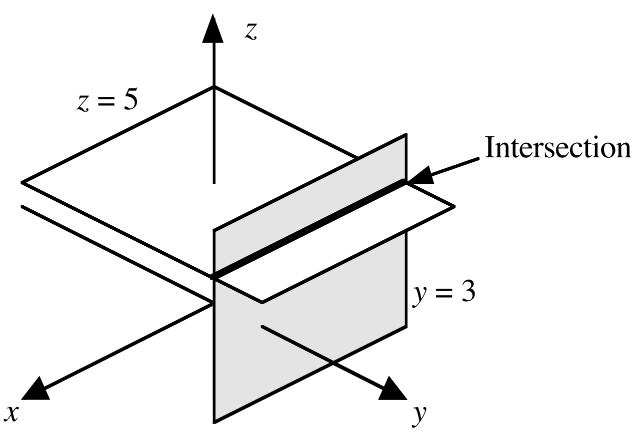
Problems from Pages 521-522 (Section 10.1)

6. **(a)** In $\mathbf{R}^2$, $x = 4$ represents a vertical line. In $\mathbf{R}^3$ this represents a plane that is parallel to the *yz* plane. The sketches are shown below.





(b) In $\mathbf{R}^3$, $y = 3$ is a plane (parallel to the *xz* plane) as is $z = 5$ (parallel to the *xy* plane). The intersection of the two is a line. The sketch is shown below.



8. The distance from the point (3, 7, −5) to each of the planes and axes are summarized in the table shown in the next page.

Part	Object	Distance
(a)	The xy-plane	5
(b)	The yz-plane	3
(c)	The xz-plane	7
(d)	The x-axis	8.60
(e)	The y-axis	5.83
(f)	The z-axis	7.62

10. The equation of the sphere is:

$$\left(x-2\right)^2+\left(y+6\right)^2+\left(z-4\right)^2=25.$$

The intersections of this sphere with each of the coordinate planes are given in the table below.

Coordinate plane	Equation of intersection with sphere
xy-plane	$\left(x-2\right)^2+\left(y+6\right)^2=9.$
xz-plane	No intersection
yz-plane	$\left(y+6\right)^2+\left(z-4\right)^2=21.$

14. Completing the square gives:

$$x^2 + y^2 + z^2 = 4x - 2y$$

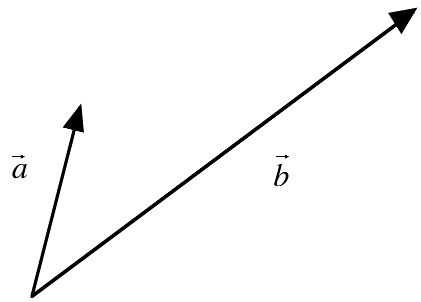
$$x^2 - 4x + y^2 + 2y + z^2 = 0$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 = 5$$

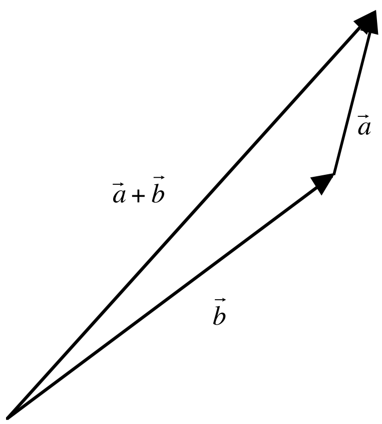
$$(x - 2)^2 + (y + 1)^2 + z^2 = 5.$$

Problems from Pages 529-530 (Section 10.2)

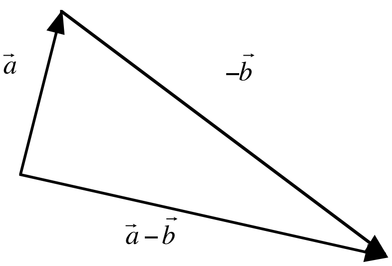
4. We will use the vectors ***a*** and ***b*** as shown below to construct the vectors sums in this problem.



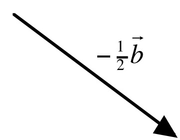
(a)



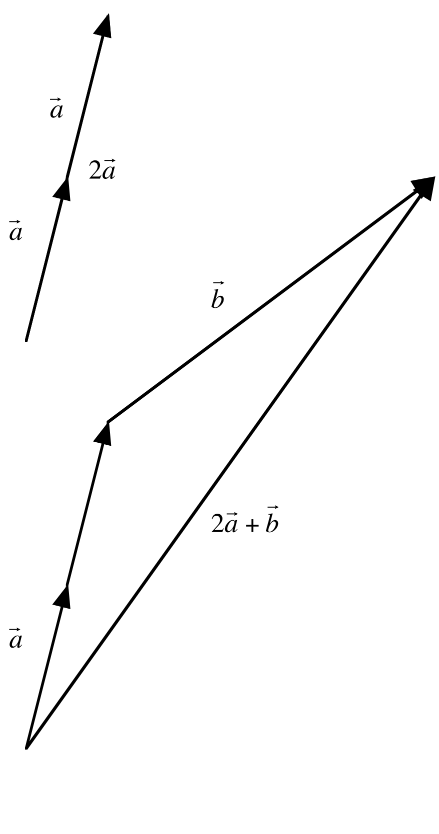
(b)



(d)

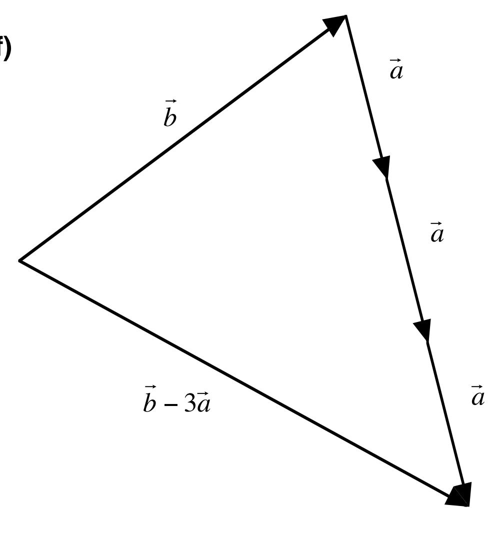


(c)



(e)

(f)



18. The vector is six times a unit vector that has the same direction as the given vector.

$$\vec{v} = \frac{6}{\sqrt{(-2)^2 + 4^2 + 2^2}}\langle -2,4,2\rangle = \frac{1}{2\sqrt{6}}\langle -2,4,2\rangle.$$

24. The tension in the wire that is three meters long is approximately <–23, 30> (where all numbers are given in units of Newtons) and the tension in the wire that is 5 meters long is approximately <23, 19>.

Problems from Pages 535-537 (Section 10.3)

10. Note that as $\vec{u}$ is a unit vector, so $|\vec{w}| = 1$ and $|\vec{v}| = \frac{\sqrt{2}}{2}$ using the Theorem of Pythagoras. The angle between ***u*** and ***w*** is $\pi/2$ radians so that these vectors are orthogonal and:

$$\vec{u} \bullet \vec{w} = 0.$$

The angle between ***u*** and ***v*** is $\pi/4$ radians so that:

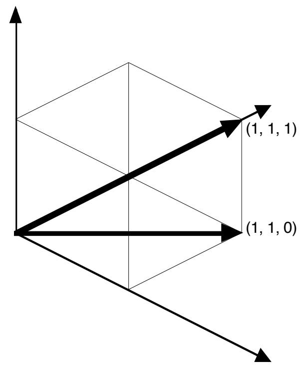
$$\vec{u} \bullet \vec{v} = (1)\cdot\left(\frac{\sqrt{2}}{2}\right)\cdot\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}.$$

20. For the vectors <–6, *b*, 2> and <*b*, b^2, *b*> to be orthogonal, the dot product of these vectors must be zero. The values of *b* that we want are the solutions of the equation:

$$b^3 - 4b = b(b + 2)(b - 2) = 0.$$

These are $b = 2$, $b = -2$ and is you consider the zero vector to be orthogonal to anything, $b = 0$.

38. We can imagine a cube with side lengths of 1 as shown in the diagram below.



The vector along the diagonal of the cube is <1, 1, 1> and the vector that lies across the diagonal of the adjacent face is <1, 1, 0>. The angle between these are given by:

$$\theta = \cos^{-1}\left(\frac{\langle 1,1,1\rangle \bullet \langle 1,1,0\rangle}{\sqrt{3}\cdot\sqrt{2}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) \approx 35^{\circ}.$$