Solutions to Homework #2

Problems from Pages 486-488 (Section 9.1)

38. **(a)** Using $\alpha = 30°$ and $v_0 = 500$ gives the pair of parametric equations:

$$x = 433.0127 \cdot t \qquad \text{and} \qquad y = 250 \cdot t - 4.9 \cdot t^2.$$

When will the bullet hit the ground?

This happens when $y = 0$. This happens at $t = 0$ and $t = 250/4.9 = 51.02$ s.

How far from the gun will it hit the ground?

$x(51.02) = 22{,}092.485$ m.

What is the maximum height attained?

$y(250/9.8) = 3188.776$ m.

(c) To eliminate t from the equations, we will rearrange the first parametric equation to make t the subject.

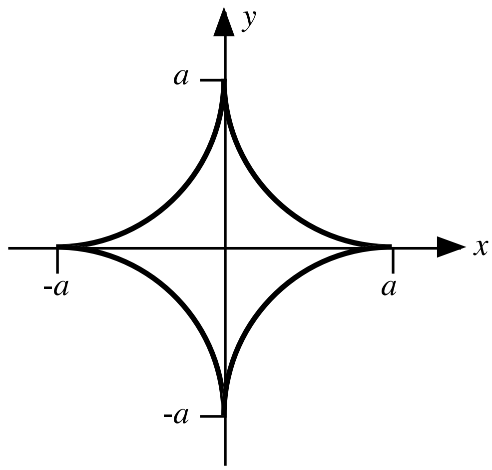
$$t = \frac{x}{v_0 \cdot \cos(\alpha)}.$$

Substituting this into the second parametric equations then shows that y is a quadratic function of x.

$$y = v_0 \cdot \sin(\alpha) \cdot \frac{1}{v_0 \cdot \cos(\alpha)} \cdot x - \frac{1}{2} g \cdot \left(\frac{1}{v_0 \cdot \cos(\alpha)}\right)^2 \cdot x^2.$$

Problems from Pages 494-495 (Section 9.2)

50. The graph of an astroid is a star-shaped curve like the one shown below. The total length of the astroid will be four times the arc length of the part of the astroid that lies in the first quadrant between $\theta = 0$ and $\theta = \pi/2$.



The length of this part of the astroid will be $\int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$. Taking the derivative of each of the parametric equations gives the following.

$$\frac{dx}{d\theta} = -3a \cdot \cos^2(\theta) \cdot \sin(\theta)$$

$$\frac{dy}{d\theta} = 3a \cdot \sin^2(\theta) \cdot \cos(\theta).$$

Squaring these and adding gives (with the use of trigonometric identities) gives the following.

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2 \cdot \sin^2(\theta) \cdot \cos^2(\theta).$$

The length of the part of the astroid we are calculating is therefore:

$$\text{Length} = \int_0^{\frac{\pi}{2}} 3a \cdot \sin(\theta) \cdot \cos(\theta) d\theta = \left[\frac{3a}{2} \sin^2(\theta)\right]_0^{\frac{\pi}{2}} = \frac{3a}{2}.$$

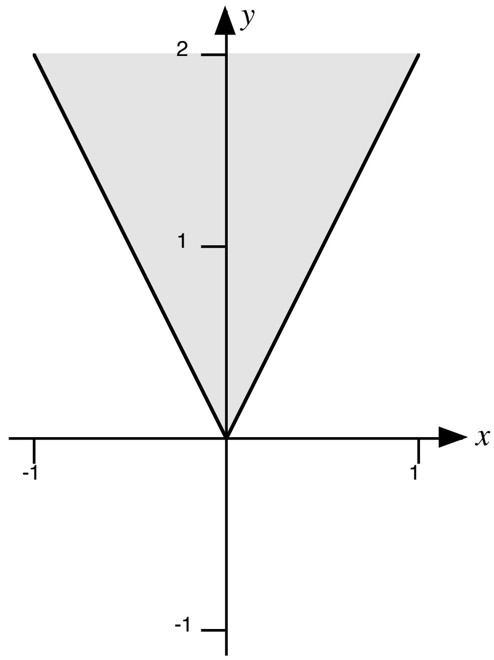
To get the total length of the astroid we multiply this by 4 to get 6*a*.

Problems from Pages 502-504 (Section 9.3)

8. The region of the *xy*-plane defined by the pair of inequalities:

$$0 \le r \qquad \text{and} \qquad \pi/3 \le \theta \le 2\pi/3$$

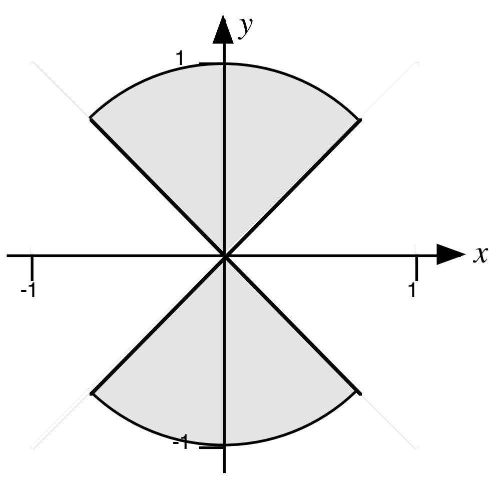
is graphed below.



12. The region of the xy-plane defined by the pair of inequalities:

$$-1 \le r \le 1 \qquad \text{and} \qquad \pi/4 \le \theta \le 3\pi/4$$

is graphed below. Remember that when radius is negative, you add π radians to the angle before plotting the point.



14. We begin by multiplying both sides of the equation by r and then rewriting in terms of x and y. This gives:

$$x^2 + y^2 = 2x + 2y.$$

Subtracting $2x$ and $2y$ from both sides and adding two to both sides of the equation gives:

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2.$$

Factoring the left hand side of the equation gives the equation of a circle with center at the point $(x, y) = (1, 1)$ and a radius of $\sqrt{2}$.

16. We begin by expressing the equation entirely in terms of $\sin(\theta)$ and $\cos(\theta)$.

$$r = \tan(\theta)\cdot\sec(\theta) = \frac{\sin(\theta)}{\cos(\theta)}\cdot\frac{1}{\cos(\theta)}.$$

Multiplying both sides of the equation by $\cos(\theta)$ gives the following.

$$r\cdot\cos(\theta) = \frac{r\cdot\sin(\theta)}{r\cdot\cos(\theta)}.$$

Rewriting everything in terms of $x = r\cdot\cos(\theta)$ and $y = r\cdot\sin(\theta)$ and rearranging to make y the subject gives $y = x^2$. This is the equation of a parabola.

46. **(a)** Graph VI.

(b) Graph III.

(c) Graph IV.

(d) Graph V.

(e) Graph II.

(f) Graph I.

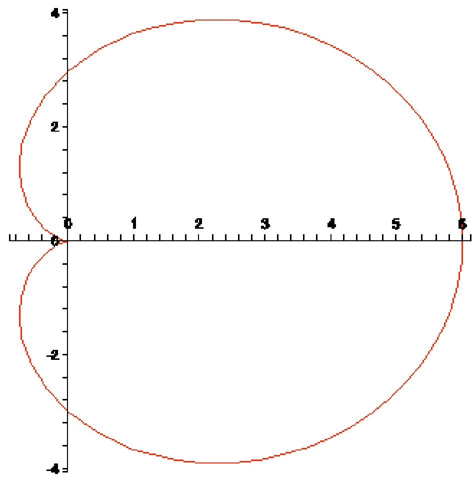
Problems from Pages 508-509 (Section 9.4)

2. Area $= \dfrac{1}{2}\displaystyle\int_{\pi}^{2\pi}\left(e^{\frac{\theta}{2}}\right)^2 d\theta = \dfrac{1}{2}\int_{\pi}^{2\pi} e^{\theta}\,d\theta = \dfrac{1}{2}\left[e^{\theta}\right]_{\pi}^{2\pi} = \dfrac{e^{2\pi} - e^{\pi}}{2}.$

6. The shaded area is the area enclosed by $r = 1 + \sin(\theta)$ between $\theta = \pi/2$ to $\theta = \pi$. This can be computed using the following integral:

$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}\left(1+\sin(\theta)\right)^2 d\theta = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}\left(1 + 2\cdot\sin(\theta) + \tfrac{1}{2}\left(1-\cos(2\theta)\right)\right)d\theta = \frac{3\pi}{8} + 1.$$

10. A graph showing the curve $r = 3(1 + \cos(\theta))$ is given below.



The area enclosed by this curve is given by the following integral.

$$\text{Area} = \frac{1}{2}\int_0^{2\pi}\left(3\cdot\left(1+\cos(\theta)\right)\right)^2 d\theta = \frac{9}{2}\int_0^{2\pi}\left(1+2\cos(\theta)+\tfrac{1}{2}\left(1+\cos(2\theta)\right)\right)d\theta = \frac{27\pi}{2}.$$