Solutions to Homework #10

Problems from Pages 680-682 (Section 12.2)

24. Volume

$$= \int_0^2 \int_0^{2y} \sqrt{4-y^2}\,dx\,dy$$

$$= \int_0^2 \left[x\sqrt{4-y^2}\right]_0^{2y} dy$$

$$= \int_0^2 2y\sqrt{4-y^2}\,dy$$

$$= \left[-\tfrac{2}{3}\left(4-y^2\right)^{3/2}\right]_0^2$$

$$= \frac{16}{3}.$$

38.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy = \int_0^1 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx$$

$$= \int_0^1 \left[y\sqrt{x^3+1}\right]_0^{x^2} dx$$

$$= \int_0^1 x^2\sqrt{x^3+1}\,dx$$

$$= \left[\tfrac{2}{9}\left(x^3+1\right)^{3/2}\right]_0^1$$

$$= \tfrac{2}{9}\left(2^{3/2}-1\right).$$

44.

$$\iint_D xy\,dA = \int_{-1}^{1} \int_{1}^{1+x^2} xy\,dy\,dx + \int_{-1}^{1} \int_{-1}^{y^2} xy\,dx\,dy$$

$$= \int_{-1}^{1} \left[\tfrac{1}{2}xy^2\right]_1^{1+x^2} dx + \int_{-1}^{1} \left[\tfrac{1}{2}x^2 y\right]_{-1}^{y^2} dy$$

$$= \int_{-1}^{1} \left(\tfrac{1}{2}x\left(1+x^2\right)^2 - \tfrac{1}{2}x\right)dx + \int_{-1}^{1} \left(\tfrac{1}{2}y^5 - \tfrac{1}{2}y\right)dy$$

$$= 0.$$

Problems from Pages 686-687 (Section 12.3)

8. $$\iint_R (x+y)\,dA = \int_{\pi/2}^{3\pi/2}\int_{1}^{2} \left(r\cos(\theta) + r\sin(\theta)\right) r\,dr\,d\theta$$

$$= \int_{\pi/2}^{3\pi/2} \left[\tfrac{1}{3}r^3\left(\cos(\theta)+\sin(\theta)\right)\right]_1^2 d\theta$$

$$= \frac{7}{3}\int_{\pi/2}^{3\pi/2} \left(\cos(\theta)+\sin(\theta)\right)d\theta$$

$$= \frac{7}{3}\left[\sin(\theta) - \cos(\theta)\right]_{\pi/2}^{3\pi/2}$$

$$= \frac{-14}{3}.$$

16. Volume $$= 2\int_{0}^{2\pi}\int_{2}^{4} \sqrt{16-r^2}\cdot r\,dr\,d\theta$$

$$= 2\int_{0}^{2\pi} \left[\tfrac{-1}{3}\left(16-r^2\right)^{3/2}\right]_2^4 d\theta$$

$$= 32\sqrt{3}\pi.$$

24. $$\int_{0}^{a}\int_{-\sqrt{a^2-y^2}}^{0} x^2 y\,dx\,dy = \int_{\pi/2}^{\pi}\int_{0}^{a} \left(r\cos(\theta)\right)^2\left(r\sin(\theta)\right) r\,dr\,d\theta$$

$$= \int_{\pi/2}^{\pi}\int_{0}^{a} r^4\cos^2(\theta)\sin(\theta)\,dr\,d\theta$$

$$= \int_{\pi/2}^{\pi} \left[\tfrac{1}{5} r^5 \cos^2(\theta)\sin(\theta)\right]_0^a d\theta$$

$$= \frac{a^5}{5} \int_{\pi/2}^{\pi} \cos^2(\theta)\sin(\theta) d\theta$$

$$= \frac{a^5}{15}.$$

Problems from Pages 700-702 (Section 12.5)

4. $$\int_0^1 \int_x^{2x} \int_0^y 2xyz\,dz\,dy\,dx = \int_0^1 \int_x^{2x} xy^3 dy\,dx$$

$$= \int_0^1 \tfrac{15}{4} x^5 dx$$

$$= \frac{5}{8}.$$

20. Volume $$= \int_0^{2\pi} \int_0^4 \left(16 - r^2\right) \cdot r \cdot dr d\theta$$

$$= \int_0^{2\pi} \left[8r^2 - \tfrac{1}{4} r^4\right]_0^4 d\theta$$

$$= 128\pi.$$

32. The five other ways of writing the iterated integral are as follows:

$$\int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x,y,z) \cdot dy \cdot dx \cdot dz$$

$$\int_0^1 \int_0^{1-y} \int_0^{1-x^2} f(x,y,z) \cdot dz \cdot dx \cdot dy$$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x^2} f(x,y,z) \cdot dz \cdot dy \cdot dx$$

$$\int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} f(x,y,z) \cdot dx \cdot dy \cdot dz + \int_0^1 \int_{1-\sqrt{1-z}}^{1} \int_0^{1-y} f(x,y,z) \cdot dx \cdot dy \cdot dz$$

$$\int_0^1 \int_0^{2y-y^2} \int_0^{1-y} f(x,y,z) \cdot dx \cdot dz \cdot dy + \int_0^1 \int_{2y-y^2}^{1} \int_0^{\sqrt{1-z}} f(x,y,z) \cdot dx \cdot dz \cdot dy$$

Problems from Pages 711-713 (Section 12.7)

26. Volume $= \int_{\pi/4}^{\pi/2} \int_0^{2\pi} \int_0^2 \rho^2 \cdot \sin(\varphi) \cdot d\rho d\theta d\varphi$

$= \dfrac{8\sqrt{2}\pi}{3}.$