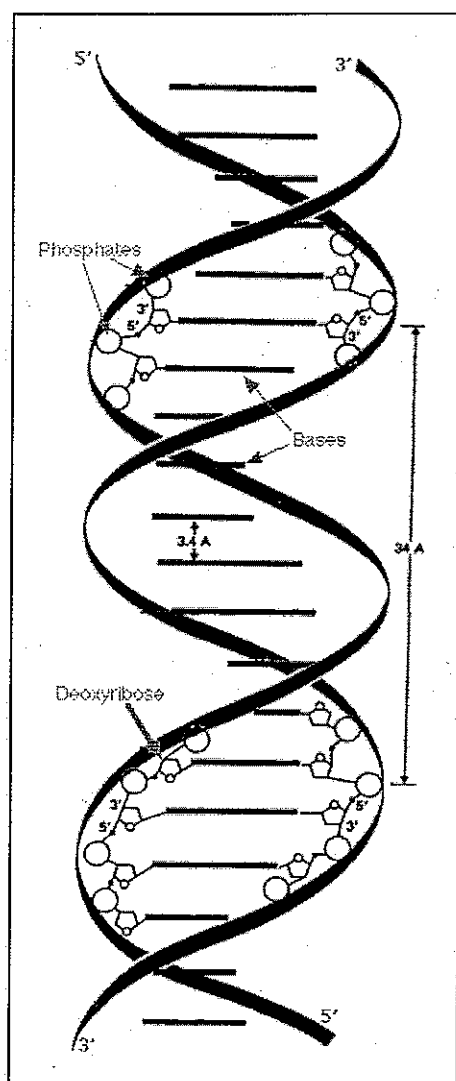


## Recitation Handout 2: Length of a DNA Molecule

The specific learning objectives for this activity are as follows:

- Create a set of parametric equations for a circle with given radius and a given interval of  $t$ -values.
- Create a set of parametric equations to represent a curve in three dimensions.
- Find the arc length of a three dimensional curve.
- Adapt the results of your calculation to correspond to one of the helices that form a typical human DNA molecule.
- Calculate the length of a human DNA molecule if it were stretched out into a straight line.

### Background: Structure of DNA



Deoxyribose nucleic acid (or DNA) is a large molecule that contains information used for the development and function of all living things and many viruses. DNA is sometimes likened to a set of blueprints for constructing a living organism.

Chemically, DNA consists of two long strands of chemicals that are coiled around each other forming a double helix (see diagram). The helical curves are joined by horizontal structures that resemble the rungs that join the two sides of a ladder.

DNA was first isolated in 1869 by the Swiss physician Friedrich Miescher from discarded surgical bandages. Miescher dubbed the substance “nuclein” and was able to conclude that it was found in the nucleus of the cell.

Evidence for DNA’s role in heredity accumulated steady after Miescher’s isolation of nuclein, but the molecular structure of DNA remained elusive. In 1953, James D. Watson and Francis Crick published an article in the scientific journal *Nature*<sup>1</sup> suggesting the double helix structure for the molecule, for which they were awarded the 1962 Nobel Prize in Medicine or Physiology (jointly with Maurice Wilkins, a New Zealand-born molecular biologist).

The history of the discovery of the structure of DNA is quite controversial and the subject of some debate. For example, many have argued that Rosalind Franklin and Raymond

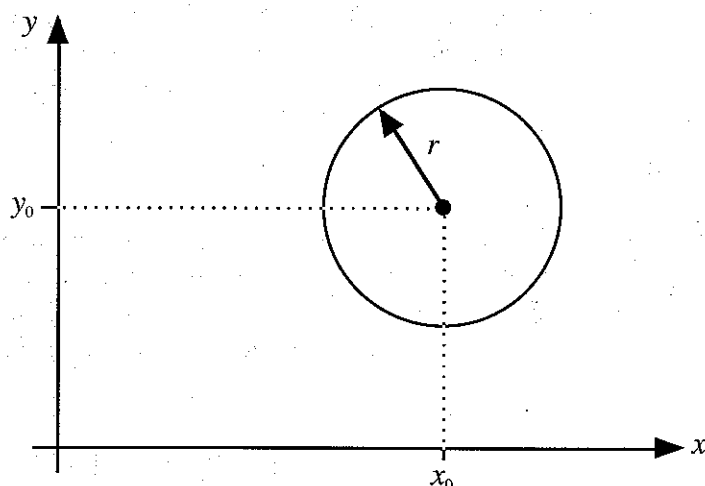
<sup>1</sup> The citation is: Watson, J. D. and Crick, F. H. C. 1953. A Structure for Deoxyribose Nucleic Acid. *Nature* **171**: 737-738. You can find this article on-line by visiting [nature.com](http://nature.com) or [jstor.org](http://jstor.org).

## SOLUTIONS

Gosling should also receive credit for the discovery of the structure of DNA. Franklin and Gosling made x-ray diffraction photographs of DNA molecules in 1951 and 1952 that Watson and Crick used to construct the double helix model, with only a passing acknowledgement of this data in their 1953 paper. (In his book, *The Double Helix: A Personal Account of the Structure of DNA*, originally published in 1968, Watson was much more candid about the use of Franklin and Gosling's work, including the admission that important data was obtained and used without the knowledge or permission of Franklin or Gosling.) These issues have taken up by several authors and historians of science, and by the popular PBS science program *NOVA* in an episode titled, "The Secret of Photo 51."<sup>2</sup>

### Calculating the Length of a DNA Molecule

1. The diagram given below shows a circle. Write down a pair of parametric equations that will trace out the whole circle once. Use  $t$  as your parameter and set up the equations so that they trace out the circle once when  $0 \leq t \leq 2\pi$ .



$$x(t) = r \cdot \cos(t) + x_0$$

$$y(t) = r \cdot \sin(t) + y_0$$

2. Suppose that instead of the interval  $0 \leq t \leq 2\pi$  you wanted to trace out the whole circle once when  $0 \leq t \leq h$ , where  $h > 0$  is a constant. Modify the parametric equations from Question 1 to accomplish this.

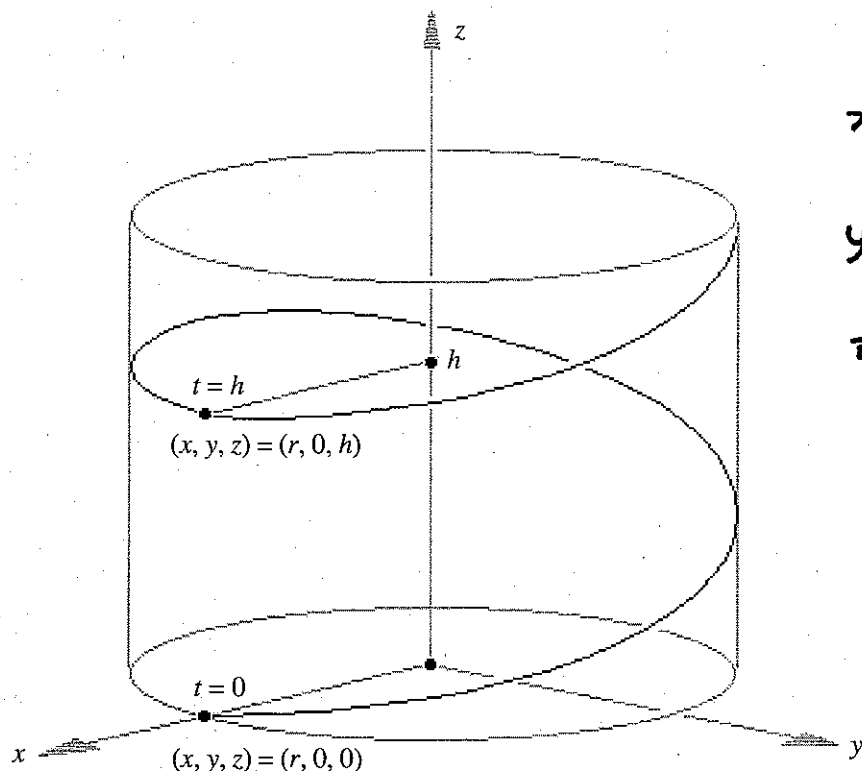
$$x(t) = r \cdot \cos\left(\frac{2\pi}{h} t\right) + x_0$$

$$y(t) = r \cdot \sin\left(\frac{2\pi}{h} t\right) + y_0$$

<sup>2</sup> You can find out more about the program by visiting: <http://www.pbs.org/wgbh/nova/photo51/>

## SOLUTIONS

3. Curves in three dimensions can be described using parametric equations. In three dimensions you need to specify three functions –  $x(t)$ ,  $y(t)$  and  $z(t)$  – that give the  $x$ ,  $y$  and  $z$  coordinates of the point on the curve at time  $t$ . The diagram given below shows a helical curve. If we assume that  $z(t) = t$ , find formulas for the functions  $x(t)$  and  $y(t)$  so that the three functions when taken together will describe the helical curve.



$$x(t) = r \cdot \cos\left(\frac{2\pi}{h} t\right)$$

$$y(t) = r \cdot \sin\left(\frac{2\pi}{h} t\right)$$

$$z(t) = t$$

4. A DNA molecule includes two helical curves joined together to form a shape that looks like a twisting ladder. Each helical curve has a radius of 10 angstroms (1 angstrom =  $10^{-10}$  meters) and rises 34 angstroms during each complete turn. Assuming that the center of one of the helical curves is the  $z$ -axis, write down a set of three parametric equations that will describe the helical part of a human DNA molecule.

Measuring  $x(t)$ ,  $y(t)$  and  $z(t)$  in units of angstroms:

$$x(t) = 10 \cdot \cos\left(\frac{2\pi}{34} t\right)$$

$$y(t) = 10 \cdot \sin\left(\frac{2\pi}{34} t\right)$$

$$z(t) = t$$

## SOLUTIONS

5. The arc length of a three dimensional parametric curve between  $t = a$  and  $t = b$  is given by:

$$\text{Arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt.$$

A human DNA molecule includes approximately  $2.9 \times 10^8$  complete turns of the helical curve. Set up an integral that will give the arc length of one of the helical curves in a human DNA molecule.

$$\frac{dx}{dt} = -\frac{20\pi}{34} \cdot \sin\left(\frac{2\pi}{34}t\right) \quad \frac{dy}{dt} = \frac{20\pi}{34} \cos\left(\frac{2\pi}{34}t\right)$$

$$\frac{dz}{dt} = 1, \quad \text{Limits of integration:} \quad \begin{aligned} a &= 0 \\ b &= 34 \times 2.9 \times 10^8 \\ &= 9.86 \times 10^9 \end{aligned}$$

$$\text{Arc length} = \int_0^{9.86 \times 10^9} \sqrt{\left(\frac{20\pi}{34}\right)^2 \cdot \sin^2\left(\frac{2\pi}{34}t\right) + \left(\frac{20\pi}{34}\right)^2 \cdot \cos^2\left(\frac{2\pi}{34}t\right) + 1} dt$$

6. Evaluate the integral from Problem 5. This will give the approximate length of a human DNA molecule if it is stretched out as far as possible to form a shape that would look like a straight ladder. Express your final answer in units of meters.

$$\text{Arc length} = \int_0^{9.86 \times 10^9} \sqrt{1 + \left(\frac{20\pi}{34}\right)^2} dt$$

$$= \left[ t \cdot \sqrt{1 + \left(\frac{20\pi}{34}\right)^2} \right]_0^{9.86 \times 10^9}$$

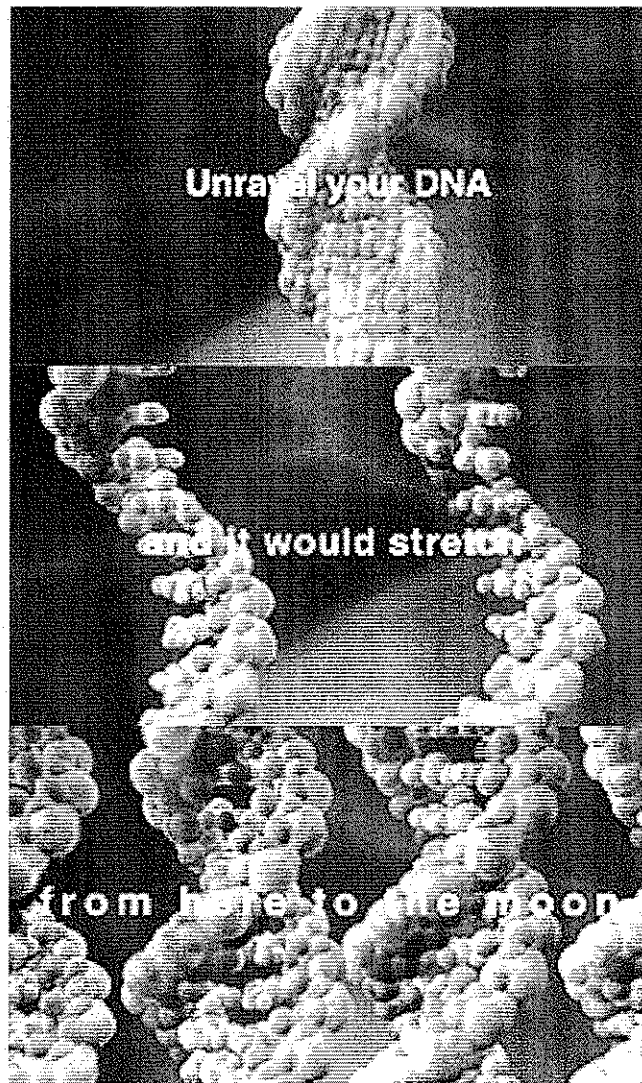
$$\doteq 2.0718 \times 10^{10} \text{ angstroms.}$$

One meter is equivalent to  $10^{10}$  angstroms, so:

$$\text{Arc length} \doteq 2.0718 \text{ meters.}$$

## SOLUTIONS

7. A few years ago, a promotional advertisement for the popular PBS science program NOVA listed various interesting scientific facts. (For example, that tigers have striped skin, not just striped fur.) One of the claims listed was that if a person could unravel all of the DNA in their body and put it together end-to-end, then it would stretch from the Earth to the Moon (a distance of approximately 384,400 km).



Human bodies contain something like  $10^{13}$  cells and each cell contains 46 DNA molecules. If you were to stretch out all the DNA in a human body and place it end-to-end, would it stretch from the Earth to the Moon? Could NOVA have made other claims in their program? (It may be helpful to bear in mind that the distance from the Earth to the Sun is approximately 149,600,000 km.)

$$\begin{aligned}\text{Total DNA length} &\approx (2.0718)(10^{13})(46) \\ &\approx 9.5302 \times 10^{14} \text{ m} \\ &= 9.5302 \times 10^{11} \text{ km.}\end{aligned}$$

This easily exceeds the distance between the Earth and the Moon. It is several thousand times the distance between the Earth and the Sun.