MATH 259 – FINAL EXAM

Friday, May 8, 2009.

NAME: ____

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A	C	D	G	H
B	F	E		

Instructions:

1. Do not separate the pages of the exam.
2. Please read the instructions for each individual question carefully.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. Be sure to use appropriate algebraic and limit notation.
7. **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

Problem	Total	Score
1	11	
2	8	
3	10	
4	10	
5	10	
6	10	
7	8	
8	7	
9	7	
10	10	
11	9	
Total	**100**	

1. 11 points.

In this problem the curve that you will be working with is the curve described by the vector function:

$$\vec{r}(t) = \langle 5 \cdot \sin(t), 5 \cdot \cos(t), 12t \rangle \qquad \text{for } t \geq 0.$$

If you give any of your answers as decimals, include at least four (4) decimal places.

(a) **(2 points)** Find the **unit** tangent vector to the curve at the point $(5, 0, 6\pi)$.

(b) **(3 points)** Find the equation of the tangent line to the curve at the point $(0, -5, 12\pi)$.

Continued on the next page.

In this problem the curve that you will be working with is the curve described by the vector function:

$$\vec{r}(t) = \langle 5 \cdot \sin(t), 5 \cdot \cos(t), 12t \rangle \qquad \text{for } t \geq 0.$$

If you give any of your answers as decimals, include at least four (4) decimal places.

(c) **(3 points)** Find the length of the curve between the points $(5, 0, 6\pi)$ and $(0, -5, 12\pi)$.

(d) **(3 points)** What are the coordinates (x, y, z) of the point that lies a distance of 26π along the curve from the point $(0, 5, 0)$?

2. 8 Points. CLEARLY INDICATE YOUR ANSWERS.

In this problem, the temperature at a point (x, y, z) is given by the function:

$$W(x,y,z) = 100 - x^2 - y^2 - z^2,$$

where temperature is measure in degrees Celsius (^{o}C) and x, y and z are all measured in meters.

(a) **(4 points)** Find the rate of change of temperature that a person would experience if they started at the point (3, −4, 5) and moved in the direction of the vector $\vec{v} = \langle 3, -4, 12 \rangle$. Include appropriate units with your answer. If you give your answer as a decimal, include at least four (4) decimal places.

(b) **(2 points)** If the person is standing at the point (3, −4, 5), in what direction should they move to experience the greatest rate of change of temperature? Give your answer in the form of a **vector**.

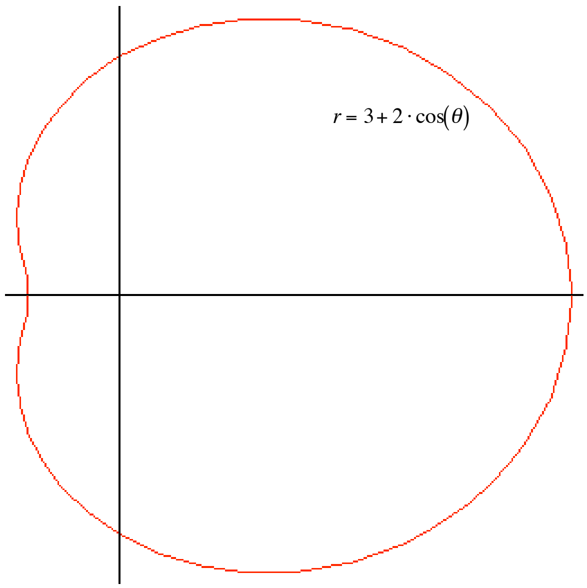
(c) **(2 points)** If the person is standing at the point (3, −4, 5), what is the greatest rate of change of temperature that they could possibly experience? Give appropriate units with your answer. If you give your answer as a decimal, include at least four (4) decimal places.

3. 10 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

Consider the curve defined by the polar equation:

$$r = 3 + 2 \cdot \cos(\theta).$$

(a) **(7 points)** Find the coordinates (x and y) of all points where the tangent line to the polar curve is **horizontal**. Find the coordinates (x and y) of **all** points where the tangent line is horizontal. If you give any of your answers as decimals, include at least four (4) decimal places. Show your work and record your results in the table at the bottom of the page. No work = no credit.



x	y

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Consider the curve defined by the parametric equations:

$$x(t) = e^{2t} + e^{-2t} \quad \text{and} \quad y(t) = 3e^{2t} - e^{-2t}.$$

(b) **(3 points)** Find the coordinates (x and y) of **all** points where the tangent line to the parametric curve is **vertical**. Find the **exact** coordinates (x and y) of **all** points where the tangent line is vertical. Show your work and record your results in the table below. No work = no credit.

x	y

4. 10 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

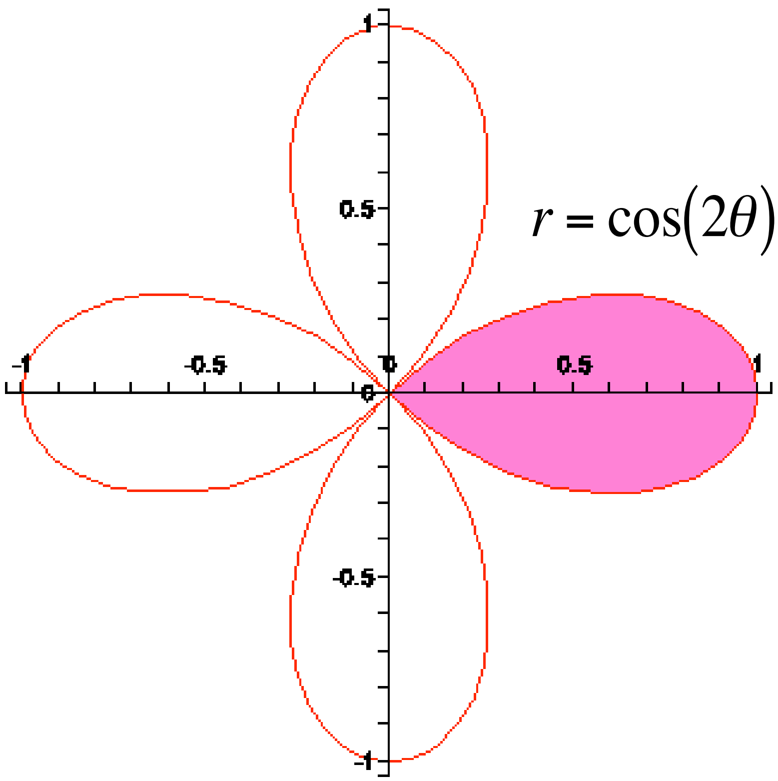
Find the **exact** area of each shaded region shown below. Show your work – no work = no credit.

You should not use your calculator on this problem for anything except evaluating functions or arithmetic. In particular, you should not use your calculator to evaluate integrals or find anti-derivatives.

You may use the following trigonometric identities without having to verify them:

$$\sin^2(x) = \tfrac{1}{2}(1 - \cos(2x)) \qquad \cos^2(x) = \tfrac{1}{2}(1 + \cos(2x)).$$

(a) (5 points)



AREA = ______________________________

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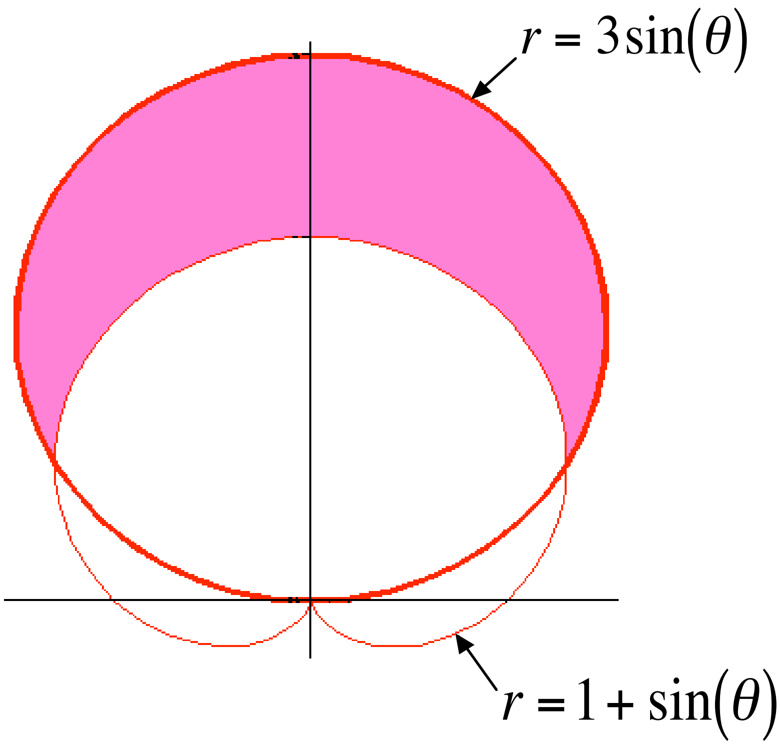
Find the area of each shaded region shown below. Show your work – no work = no credit.

You should not use your calculator on this problem for anything except evaluating functions or arithmetic. In particular, you should not use your calculator to evaluate integrals or find anti-derivatives.

You may use the following trigonometric identities without having to verify them:

$$\sin^2(x) = \tfrac{1}{2}(1 - \cos(2x)) \qquad \cos^2(x) = \tfrac{1}{2}(1 + \cos(2x)).$$

(b) **(5 points)**



AREA = ______________________________

5. 10 points total. SHOW YOUR WORK.

In this problem the function $f(x, y)$ will always refer to the function defined by the formula:

$$f(x,y) = x^2 + y^2 - x - y + 1.$$

(a) **(4 points)** Find the x and y coordinates of any critical points of $f(x, y)$. Record your results in the table below.

(b) **(2 points)** Classify the critical points that you found in Part (a) as local maximums, local minimums or saddle points. Record your results in the table below.

RECORD RESULTS FOR PARTS (A) AND (B) IN THIS TABLE:

x	y	Classification	$f(x, y)$

(c) **(4 points)** Find the global maximum and global minimum of $f(x, y)$ on the disk where $x^2 + y^2 \leq 1$. Show your work and write your final answers in the spaces provided below. If you give your answers as decimals, include at least four (4) decimal places.

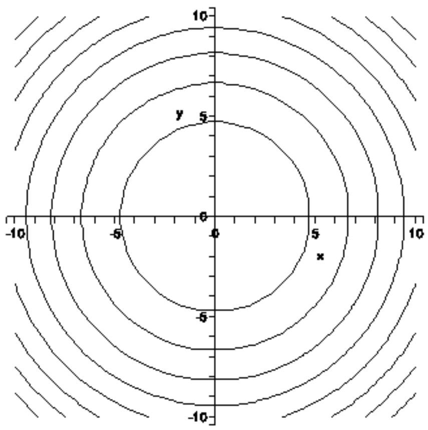
MAXIMUM VALUE OF $f(x, y)$: ____________________

MINIMUM VALUE OF $f(x, y)$: ____________________

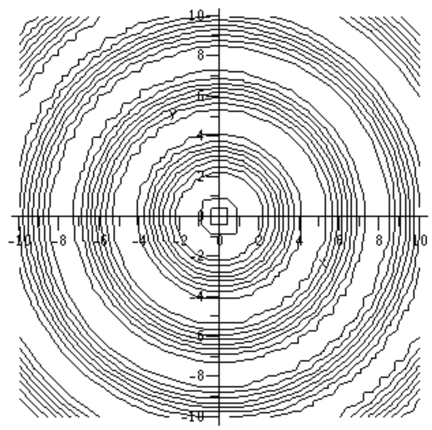
6. 10 points total. 2 POINTS FOR EACH CORRECT MATCH.

Match the contour plots with the functions given below. If you do not think that any of the contour plots does a good job of showing the level curves of a particular function, write the word **NONE** next to that function.

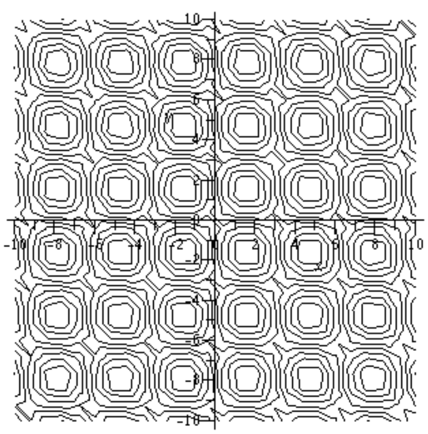
CONTOUR PLOT I



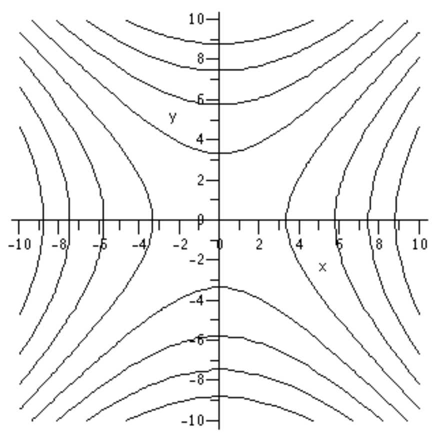
CONTOUR PLOT II



CONTOUR PLOT III



CONTOUR PLOT IV



(a) $f(x,y) = \sin(\sqrt{x^2 + y^2})$ **CONTOUR PLOT = __________________**

(b) $f(x,y) = \dfrac{1}{\sin(x)}$ **CONTOUR PLOT = __________________**

(c) $f(x,y) = 4 - x^2 - y^2$ **CONTOUR PLOT = __________________**

(d) $f(x,y) = x^2 - y^2$ **CONTOUR PLOT = __________________**

(e) $f(x,y) = \sin(x) \cdot \sin(y)$ **CONTOUR PLOT = __________________**

7. **8 Points. SHOW YOUR WORK.**

(a) **(4 points)** Find an equation for the plane that includes both the point (1, 3, 0) and the line

$$\langle x, y, z\rangle = \langle 2,7,1\rangle + t\cdot\langle -1,1,1\rangle.$$

Show all of your work and express your final answer in the form: $ax + by + cz = d$.

(b) **(4 points)** Let $f(x, y, z) = xyz$ and $g(x, y, z) = x^2 + y^2 - z$. Find a three-dimensional **unit vector** $\vec{u}$ that makes both of the directional derivatives equal zero:

$$D_{\vec{u}} f(1,1,1) = 0 \quad \text{and} \quad D_{\vec{u}} g(1,1,1) = 0.$$

8. 7 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.

Use Lagrange Multipliers to find the maximum and minimum values of the function:

$$f(x,y,z) = x - 2y - z$$

subject to the constraint:

$$x^2 + 3y^2 + z^2 = 1.$$

Show your work and record your answers in the space provided below.

MAXIMUM VALUE OF *f*(*x*, *y*, *z*): ______________________________

MINIMUM VALUE OF *f*(*x*, *y*, *z*): ______________________________

9. 7 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.

Evaluate the surface integral:

$$\iint_S f(x,y,z)\,dS,$$

where $f(x, y, z) = 1$ and S is the surface parametrized by:

$$r(\theta,\varphi) = < (b + a\cdot\cos(\varphi))\cdot\cos(\theta), (b + a\cdot\cos(\varphi))\cdot\sin(\theta), a\cdot\sin(\varphi) >,$$

where a and b are positive constants with $0 < a < b$ and $0 \le \theta \le 2\pi$ and $0 \le \varphi \le 2\pi$.

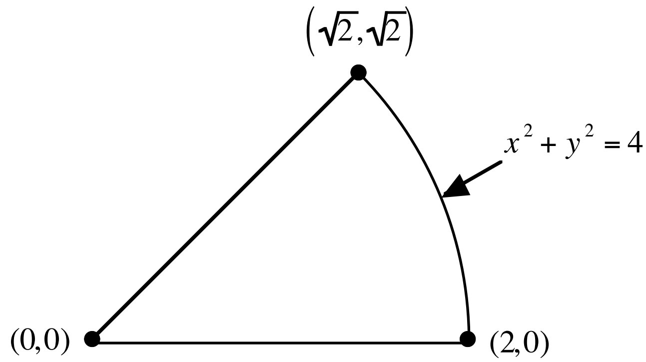
You should not use your calculator on this problem for anything except evaluating functions or arithmetic. In particular, you should not use your calculator to evaluate integrals or find anti-derivatives.

10. 10 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.

You should not use your calculator on this problem for anything except evaluating functions or arithmetic. In particular, you should not use your calculator to evaluate integrals or find anti-derivatives.

(a) **(5 points)** Let C be the path shown in the diagram given below. Calculate the value of the path integral:

$$\int_C \left(y^2 - x^2 y\right) \cdot dy + xy^2 \cdot dx .$$



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You should not use your calculator on this problem for anything except evaluating functions or arithmetic. In particular, you should not use your calculator to evaluate integrals or find anti-derivatives.

(b) **(5 points)** Let S be the surface of the sphere $x^2 + y^2 + z^2 = a^2$, where a is a positive constant. Let $\vec{F}(x,y,z) = \langle x^3, y^3, z^3 \rangle$. Calculate the value of the surface integral:

$$\iint_S \vec{F} \bullet d\vec{S}.$$

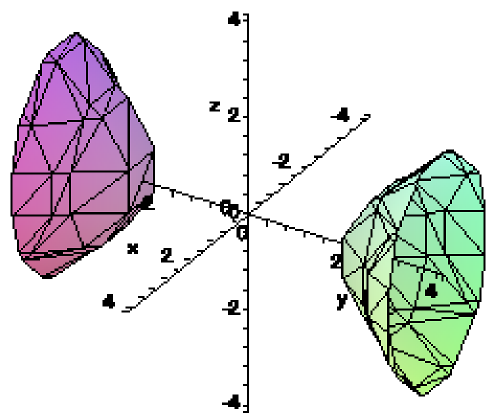
11. 9 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

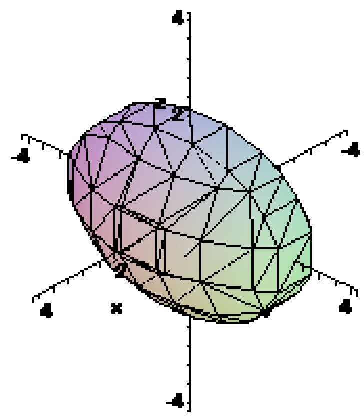
In this problem you will be interested in the surface defined by the equation:

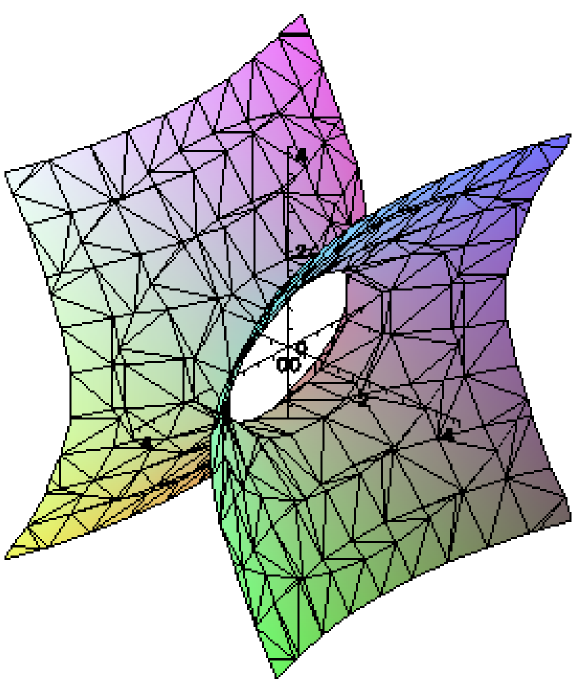
$$4x^2 - y^2 + 2z^2 + 4 = 0.$$

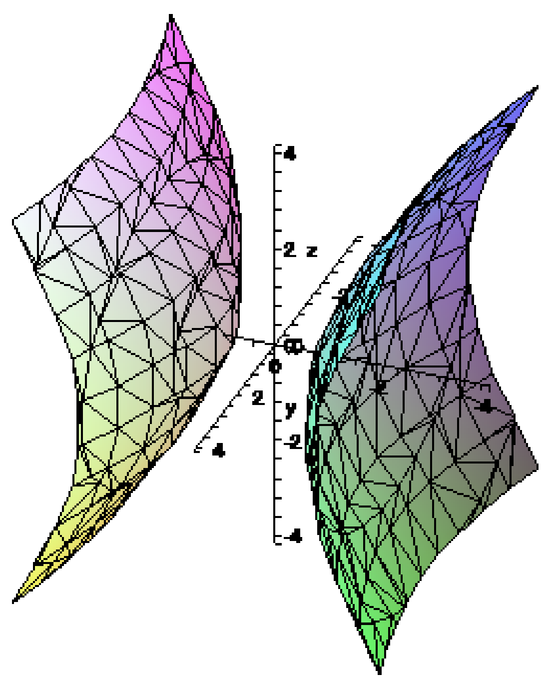
(a) **(2 points)** Classify the surface (i.e. is it a plane, ellipsoid, hyperboloid of two sheets, etc.?)

(b) **(2 points)** Which graph (of those shown below) does the best job of showing the graph of the surface defined by the equation given above? CIRCLE ONE (AND ONLY ONE) GRAPH.









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SHOW YOUR WORK. NO WORK = NO CREDIT.

In this problem you will be interested in the surface defined by the equation:

$$4x^2 - y^2 + 2z^2 + 4 = 0.$$

(c) **(5 points)** Find an equation for the tangent plane to the surface based at the point:

$$(x, y, z) = (1, 4, 2).$$