SOLUTIONS

Quiz #9

1. Determine the convergence or divergence of each of the following series. If you do not justify your answer, you will get zero credit, even if you circle the correct final answer.

In each case, demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test.* Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work and how the convergence test justifies your answer. As the final part of your answer in each part, CIRCLE either CONVERGES or DIVERGES.

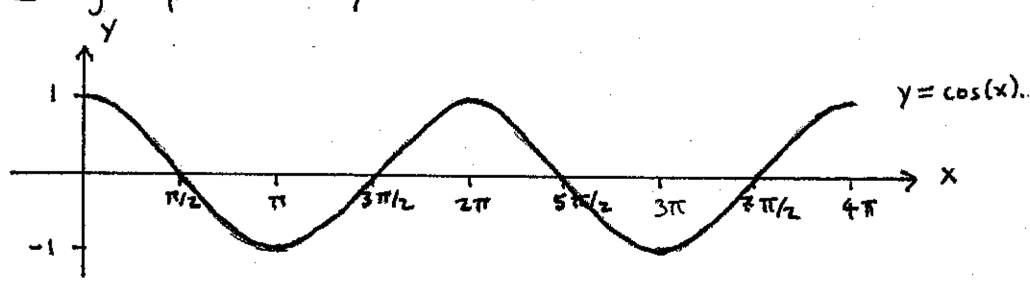
(a) **(3 points)** $$\sum_{n=1}^{\infty} \frac{3+2\cdot\cos(n)}{n}$$

JUSTIFICATION:

Test used: Comparison Test.

Note that $\sum_{n=1}^{\infty} \frac{1}{n}$ is a p-series with $p \leq 1$, so it diverges.

The graph of $y = \cos(x)$ is shown below.



The graph shows that when $n > 0$,

$$\cos(n) \geq -1$$
$$2\cos(n) \geq -2$$
$$3 + 2\cdot\cos(n) \geq 1$$
$$\frac{3+2\cos(n)}{n} \geq \frac{1}{n} > 0$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the Comparison Test gives that $\sum_{n=1}^{\infty} \frac{3+2\cos(n)}{n}$ also diverges.

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

SOLUTIONS

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(b) **(3 points)** $$\sum_{n=2}^{\infty} \frac{n^2 \cdot e^n}{n!}$$

JUSTIFICATION:

Test used: Ratio test.

$$a_n = \frac{n^2 \cdot e^n}{n!} \qquad a_{n+1} = \frac{(n+1)^2 e^{n+1}}{(n+1)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2 e^{n+1}}{(n+1)!} \cdot \frac{n!}{n^2 e^n} = \frac{(n+1)^2}{n^2} \cdot \frac{e}{(n+1)}$$

$$= \frac{e(n+1)}{n^2}$$

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{e(n+1)}{n^2} = 0.$$

Since $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 < 1$, the Ratio test gives that the infinite series $\sum_{n=2}^{\infty} \frac{n^2 \cdot e^n}{n!}$ converges.

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

SOLUTIONS.

2. Suppose that all you knew about the hyperbolic sine ($\sinh(x)$) and hyperbolic cosine ($\cosh(x)$) functions was that they had the following properties:

- $\sinh(0) = 0$
- $\cosh(0) = 1$
- $\frac{d}{dx}(\sinh(x)) = \cosh(x)$
- $\frac{d}{dx}(\cosh(x)) = \sinh(x)$

(a) **(2 points)** Use the information provided, derivatives and the definition of a Taylor polynomial above to find the degree 7 Taylor polynomial for $\sinh(x)$ based at the anchor point $a = 0$.

Let $f(x) = \sinh(x)$. Then:

$f(0) = 0 \quad f'(0) = 1 \quad f''(0) = 0 \quad f'''(0) = 1$

$f^{(iv)}(0) = 0 \quad f^{(v)}(0) = 1 \quad f^{(vi)}(0) = 0 \quad f^{(vii)}(0) = 1.$

The Taylor polynomial is given by:

$$P_7(x) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7$$

(b) **(2 points)** Based on the patterns that you can see in the Taylor polynomial from Part (a), what do you think the Taylor series for $\sinh(x)$ (based at the anchor point $a = 0$) will be? Express your answer in sigma (Σ) notation.

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1}$$

There are many other valid ways to express this Taylor series.

$$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} x^{2n-1}$$