SOLUTIONS

Math 122 **Fall 2008**

Quiz #8

1. Determine the convergence or divergence of each of the following series. If you do not justify your answer, you will get zero credit, even if you circle the correct final answer.

In each case, demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test.* Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work and how the convergence test justifies your answer. As the final part of your answer in each part, CIRCLE either CONVERGES or DIVERGES.

(a) **(3 points)** $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$

JUSTIFICATION:

This problem can be solved using the Ratio Test.

$$a_n = \frac{3^n}{(2n)!} \qquad a_{n+1} = \frac{3^{n+1}}{(2n+2)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{3^n} = \frac{3}{(2n+2)(2n+1)}.$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{3}{(2n+2)(2n+1)} = 0.$$

Since $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, the ratio test gives that $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$ converges.

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

SOLUTIONS

Determine the convergence or divergence of each of the following series. If you do not justify your answer, you will get zero credit, even if you circle the correct final answer.

In each case, demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test.* Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work and how the convergence test justifies your answer. As the final part of your answer in each part, CIRCLE either CONVERGES or DIVERGES.

(b) **(3 points)** $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(n^2)}$

JUSTIFICATION:

This problem can be solved using the comparison test.

Note that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2 \cdot n}$ is a constant multiple of the p-series with $p=1$. This series diverges.

For $n > 1$:

$$n < e^n$$
$$\ln(n) < n$$
$$2 \cdot \ln(n) < 2 \cdot n$$
$$\ln(n^2) < 2 \cdot n$$
$$\frac{1}{\ln(n^2)} > \frac{1}{2n}.$$

Since $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2n}$ diverges an $\dfrac{1}{\ln(n^2)} > \dfrac{1}{2n}$, the Comparison test gives that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(n^2)}$ diverges.

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

SOLUTIONS

2. Suppose that that the N^{th} partial sum of the infinite series, $\sum_{k=0}^{\infty} a_k$, is given by the formula:

$$S_N = \frac{16 - \frac{1}{N}}{5 + \frac{1}{N^2}}.$$

(a) **(2 points)** Does the series $\sum_{k=0}^{\infty} a_k$ converge or not? If so, briefly explain why and calculate the sum of the series. If not, briefly explain why not.

Yes, the series does converge. This is because the limit of the formula for S_N (as $N \to \infty$) exists and is finite.

$$\lim_{N \to \infty} S_N = \lim_{N \to \infty} \frac{16 - 1/N}{5 + 1/N^2} = \frac{16}{5}.$$

(b) **(2 points)** Given what you know about S_N, is it possible to calculate the following limit:

$$\lim_{k \to \infty} a_k \quad ?$$

If so, calculate the limit. If not, briefly explain why not.

We know that $\lim_{N \to \infty} S_N$ exists and is finite, so we know that the infinite series $\sum_{k=0}^{\infty} a_k$ converges.

The statement (nth term test for divergence):

$$\lim_{k \to \infty} a_k \neq 0 \Rightarrow \sum_{k=0}^{\infty} a_k \text{ diverges}$$

is logically equivalent to:

$$\sum_{k=0}^{\infty} a_k \text{ converges} \Rightarrow \lim_{k \to \infty} a_k = 0.$$

So, it is possible to determine the limit, and the value of the limit is zero.