SOLUTIONS

Math 122 **Fall 2008**

Quiz #7

For each question, be careful to indicate your final answer **and where appropriate, show how you obtained it.**

1. **(5 points)** Find the formula of the solution, $y(x)$, of the following initial value problem:

$$y'' + 6y' + 10y = x \qquad y(0) = 0 \qquad y'(0) = 0.$$

Show your work and write your final answer in the space provided on the next page.

Step 1: Solve the homogeneous equation.

$$y'' + 6y' + 10y = 0$$

Char. Eq^n: $r^2 + 6r + 10 = 0$

Roots: $r = \dfrac{-6 \pm i\sqrt{40 - 36}}{2}$

$$r = -3 \pm i \cdot 1$$

The solution of the homogeneous equation is:

$$y_h(x) = C_1 e^{-3x} \cdot \cos(x) + C_2 \cdot e^{-3x} \cdot \sin(x).$$

Step 2: Create a particular solution.

Function & Derivatives	What's important.
$N(x) = x$	x
$N'(x) = 1$	1.
$N''(x) = 0$	

The particular solution will resemble:

$$y_p(x) = F \cdot x + G$$

where F and G are constants.

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ADDITIONAL SPACE FOR PROBLEM #1.

$$y'' + 6y' + 10y = x \qquad y(0) = 0 \qquad y'(0) = 0.$$

To determine F and G, plug $y_p(x)$ into the nonhomogeneous differential equation.

$$y_p''(x) + 6\,y_p'(x) + 10\,y_p(x) = x$$

$$6F + 10Fx + 10G = x$$

Equating coefficients of powers of x:

x^1: $10F = 1$

x^0: $6F + 10G = 0$

gives $F = 1/10$, $G = -6/100$ and $y_p(x) = \frac{1}{10}x - \frac{6}{100}$.

<u>Step 3:</u> Use initial values.

$$y(x) = y_h(x) + y_p(x) = C_1 e^{-3x}\cos(x) + C_2 e^{-3x}\sin(x) + 1/10\,x - 6/100.$$

$y(0) = 0$: $C_1 - 6/100 = 0$ so $C_1 = 6/100$

$y'(0) = 0$: $-3C_1 + C_2 + 1/10 = 0$ so $C_2 = 8/100$.

FINAL ANSWER: $y(x) = \frac{6}{100}e^{-3x}\cos(x) + \frac{8}{100}e^{-3x}\sin(x) + \frac{1}{10}x - \frac{6}{100}$.

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2. Magnesium sulfate (commonly known as Epsom salt) is sometimes given intravenously to people who have severe asthma attacks. The magnesium sulfate helps the bronchi in the lungs to relax and the person to breathe more easily.

A person is admitted to a hospital emergency room at time $t = 0$ with no magnesium sulfate in their body. The person is experiencing a severe asthma attack and is immediately given magnesium sulfate intravenously at a rate of 1.2 g per hour. Magnesium sulfate is removed from the body at a rate proportional to the mass of magnesium sulfate in the body. The constant of proportionality is 0.133.

(a) **(2 points)** Let $M(t)$ represent the mass of magnesium sulfate in the patient's body after t hours. Write down a differential equation and an initial condition that define $M(t)$.

$$\frac{dM}{dt} = 1.2 - 0.133 \cdot M(t)$$

$$M(0) = 0$$

(b) **(2 points)** Use the differential equation and initial value from Part (a) to find a formula for $M(t)$. Show your work and clearly indicate your final answer.

$$\frac{dM}{dt} = -0.133 \cdot (M - 9.0226)$$

$$\int \frac{1}{M - 9.0226}\, dM = \int -0.133 \cdot dt$$

$$\ln(|M - 9.0226|) = -0.133t + C$$

$$M = 9.0226 - A e^{-0.133t}$$

where $A = \pm e^{C}$.

Additional space is available on the next page.

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To determine the value of the constant A, use $M(0) = 0$.

$$0 = 9.0226 - A \cdot e^{0}$$

$$A = 9.0226.$$

Final answer:

$$M(t) = 9.0226 - 9.0226\, e^{-0.133t}$$

(c) **(1 point)** The toxicity of magnesium sulfate in humans is not very clearly established. For the sake of this problem we will assume a 50% chance of dying from magnesium sulfate poisoning if 8g of magnesium sulfate accumulates in the patient's body. How many hours does it take for this much magnesium sulfate to accumulate in the patient's body?

Set $M(t) = 8$ and solve for t.

$$9.0226 - 9.0226\, e^{-0.133t} = 8$$

$$9.0226\, e^{-0.133t} = 9.0226 - 8$$

$$e^{-0.133t} = \frac{9.0226 - 8}{9.0226}$$

$$t = \frac{-1}{0.133} \ln\left(\frac{9.0226 - 8}{9.0226}\right)$$

$$\approx 16.37 \text{ hours.}$$